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A Note on Characterization of Intuitionistic Fuzzy Ideals in Γ- Near-Rings

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Abstract

In this paper, we study some properties of intuitionistic fuzzy ideals of a Γ–near– ring and prove some results on these.

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Introduction

The notion of a fuzzy set was introduced by L.A.Zadeh[10], and since then this concept have been applied to various algebraic structures. The idea of "Intuitionistic Fuzzy Set" was first published by K.T.Atanassov[1] as a generalization of the notion of fuzzy set. Γ- near-rings were defined by Bh.Satyanarayana [9] and G.L.Booth [2, 3] studied the ideal theory in Γ-near-rings. W. Liu[7] introduced fuzzy ideals and it has been studied by several authors. The notion of fuzzy ideals and its properties were applied to semi groups, BCK- algebras and semi rings. Y.B. Jun [5, 6] introduced the notion of fuzzy left (resp.right) ideals. In this paper, we introduce the notion of intuitionistic fuzzy ideals in Γ–near- rings and study some of its properties.

Preliminaries

In this section we include some elementary aspects that are necessary for this paper.

Definition 2.1 A non–empty set R with two binary operations "+" (addition) and "." (multiplication) is called a near-ring if it satisfies the following axioms:

i. (R, +) is a group,
ii. (R, .) is a semigroup,
iii. $(x + y) \cdot z = x \cdot z + y \cdot z$, for all $x, y, z \in R$. It is a right near-ring because it satisfies the right distributive law.

Definition 2.2 A Γ-near-ring is a triple $(M, +, \Gamma)$ where

i. (M, +) is a group,
ii. is a nonempty set of binary operators on M such that for each $\alpha \in \Gamma$, $(M, +, \alpha)$ is a near –ring,
iii. $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Definition 2.3 A subset A of a Γ-near-ring M is called a left (resp. right) ideal of M if

i. (A, +) is a normal divisor of (M, +),
ii. $u\alpha(x + v) - u\alpha v \in A$ (resp . $x\alpha u \in A$) for all $x \in A$, $\alpha \in \Gamma$ and $u, v \in M$.

Definition 2.4 A fuzzy set μ in a Γ-near-ring M is called a fuzzy left (resp.right) ideal of M if

i. $\mu(x-y) \geq \min\{ \mu(x), \mu(y) \}$,
ii. $\mu(y + x-y) \geq \mu(x)$, for all $x, y \in M$.
iii. $\mu(u\alpha(x + v) - u\alpha v) \geq \mu(x)$ (resp. $\mu(x\alpha u) \geq \mu(x)$) for all $x, u, v \in M$ and $\alpha \in \Gamma$.

Definition 2.5 [1] Let X be a nonempty fixed set .An intuitionistic fuzzy set (IFS) A in X is an object having the form $A = \{ < x, \mu_A(x) , \nu_A(x) >/ x\in X \}$, where the functions $\mu_A : X \to [0, 1]$ and $\nu_A: X \to [0, 1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Notation. For the sake of simplicity, we shall use the symbol $A= < \mu_A, \nu_A >$ for the IFS $A = \{< x, \mu_A(x) , \nu_A(x) > /x\in X \}$.

Definition 2.6 [1]. Let X be a non-empty set and let $A = < \mu_A ,\nu_A >$ and $B = < \mu_B ,\nu_B >$ be IFSs in X. Then

1. $A \subset B$ iff $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$.
2. $A = B$ iff $A \subset B$ and $B \subset A$.
3. $A^c = < \nu_A, \mu_A >$.
4. $A \cap B = (\mu_A \wedge \mu_B , \nu_A \vee \nu_B)$.
5. $A \cup B = (\mu_A \vee \mu_B , \nu_A \wedge \nu_B)$.
6. $\Box A = (\mu_A ,1-\mu_A)$, $\Diamond A = (1-\nu_A , \nu_A)$.

Definition 2.7. Let μ and ν be two fuzzy sets in a Γ-near-ring For $s, t \in[0, 1]$ the set $U(\mu,s) = \{ x\in\mu(x) \geq s \}$ is called upper level of μ. The set $L(\nu,t)=\{x\in\nu(x) \leq t \}$ is called lower level of ν.

Definition 2.8. Let A be an IFS in a Γ− ring M. For each pair < t, s > ∈ [0, 1] with t + s ≤ 1, the set $A_{<t,s>}$ = { x ∈ X / $\mu_A(x) \geq$ t and $\nu_A(x) \leq$ s } is called a <t, s> − level subset of A .

Definition 2.9. Let A = < μ_A , ν_A > be an intuitionistic fuzzy set in M and let t∈[0, 1]. Then the sets U(μ_A ; t) = {x∈M : $\mu_A(x) \geq t$} and L(ν_A ; t) = {x∈M : $\nu_A(x) \leq t$} are called upper level set and lower level set of A respectively.

Intuitionistic fuzzy ideals

In what follows, let M denote a Γ–near-ring unless otherwise specified.

Definition 3.1. An IFS A = < μ_A , ν_A > in M is called an intuitionistic fuzzy left (resp. right) ideal of a Γ – near-ring M if

1. $\mu_A(x - y) \geq \{ \mu_A(x) \wedge \mu_A(y) \}$,
2. $\mu_A(y + x - y) \geq \mu_A(x)$,
3. $\mu_A(u\alpha(x+v)-u\alpha v) \geq \mu_A(x)$ (resp. $\mu_A(x\alpha u) \geq \mu_A(x)$),
4. $\nu_A(x - y) \leq \{ \nu_A(x) \vee \nu_A(y) \}$,
5. $\nu_A(y+x - y) \leq \nu_A(x)$,
6. $\nu_A(u\alpha(x+v)-u\alpha v) \leq \nu_A(x)$ (resp. $\nu_A(x\alpha u) \leq \nu_A(x)$),

for all x, y, u, v ∈ M and α ∈ Γ.

Example 3.2. Let R be the set of all integers then R is a ring.

Take M = Γ = R. Let a, b ∈ M, α ∈ Γ, suppose aαb is the product of a , α , b ∈ R.

Then M is a Γ- near-ring.

Define an IFS A = < μ_A ,ν_A > in R as follows.

$\mu_A(0) = 1$ and $\mu_A(\pm1) = \mu_A(\pm2) = \mu_A(\pm3) = ... = t$ and $\nu_A(0) = 0$ and $\nu_A(\pm1) = \nu_A(\pm2) = \nu_A(\pm3) = ... = s$, where t ∈ [0, 1] , s ∈ [0, 1] and t + s ≤ 1.

By routine calculations, clearly A is an intuitionistic fuzzy ideal of a Γ- near-ring R.

Theorem 3.3. A is an ideal of a Γ–near-ring M if and only if Ã = < $\mu_{\tilde{A}}$, $\nu_{\tilde{A}}$ > where

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 & x \in A \\ 0 & Otherwise \end{cases} \quad \nu_{\tilde{A}}(x) = \begin{cases} 0 & x \in A \\ 1 & Otherwise \end{cases}$$

is an intuitionistic fuzzy left (resp.right) ideal of M.

Proof (⇒) : Let A be a left (resp.right) ideal of M.
Let x , y, u, v ∈ M and α ∈ Γ.

If x , y ∈ A, then x – y ∈ A, y + x – y ∈ A and (uα(x +v) -uαv) ∈ A. Therefore

$\mu_{\tilde{A}}(x-y) = 1 \geq \{ \mu_{\tilde{A}}(x) \wedge \mu_{\tilde{A}}(y) \}, \mu_{\tilde{A}}(y + x - y) = 1 \geq \mu_{\tilde{A}}(x)$ and
$\mu_{\tilde{A}}(u\alpha(x+v) - u\alpha v) = 1 = \mu_{\tilde{A}}(x)$ (resp. $\mu_{\tilde{A}}(x\alpha u) = \mu_{\tilde{A}}(x) = 1$),
$\nu_{\tilde{A}}(x-y) = 0 \leq \{\nu_{\tilde{A}}(x) \vee \nu_{\tilde{A}}(y) \}, \nu_{\tilde{A}}(y + x - y) = 0 \leq \nu_{\tilde{A}}(x)$ and
$\nu_{\tilde{A}}(u\alpha(x+v) - u\alpha v) = 0 = \nu_{\tilde{A}}(x)$ (resp. $\nu_{\tilde{A}}(x\alpha u) = \nu_{\tilde{A}}(x) = 0$).

If $x \notin A$ or $y \notin A$ then $\mu_{\tilde{A}}(x) = 0$ or $\mu_{\tilde{A}}(y) = 0$.
Thus we have

$\mu_{\tilde{A}}(x-y) \geq \{ \mu_{\tilde{A}}(x) \wedge \mu_{\tilde{A}}(y) \}, \mu_{\tilde{A}}(y + x - y) \geq \mu_{\tilde{A}}(x)$ and
$\mu_{\tilde{A}}(u\alpha(x+v) - u\alpha v) \geq \mu_{\tilde{A}}(x)$ (resp. $\mu_{\tilde{A}}(x\alpha u) \geq \mu_{\tilde{A}}(x)$),
$\nu_{\tilde{A}}(x-y) \leq \{\nu_{\tilde{A}}(x) \vee \nu_{\tilde{A}}(y) \}, \nu_{\tilde{A}}(y + x - y) \leq \nu_{\tilde{A}}(x)$ and
$\nu_{\tilde{A}}(u\alpha(x+v) - u\alpha v) \leq \nu_{\tilde{A}}(x)$ (resp. $\nu_{\tilde{A}}(x\alpha u) \leq \nu_{\tilde{A}}(x)$).

Hence $\tilde{A}$ is an intuitionistic fuzzy left (resp.right) ideal of M.
($\Leftarrow$) : Let $\tilde{A}$ be an intuitionistic fuzzy left (resp.right) ideal of M.
Let $x, y \in M$ and $\alpha \in \Gamma$.

If $x, y, u, v \in A$, then

$\mu_{\tilde{A}}(x-y) \geq \{ \mu_{\tilde{A}}(x) \wedge \mu_{\tilde{A}}(y)\} = 1$
$\nu_{\tilde{A}}(x-y) \leq \{\nu_{\tilde{A}}(x) \vee \nu_{\tilde{A}}(y)\} = 0$

So $x-y \in A$.

$\mu_{\tilde{A}}(y + x - y) \geq \mu_{\tilde{A}}(x) = 1$
$\nu_{\tilde{A}}(y + x - y) \leq \nu_{\tilde{A}}(x) = 0$

So $(y + x - y) \in A$.
Also

$\mu_{\tilde{A}}(u\alpha(x+v) - u\alpha v) \geq \mu_{\tilde{A}}(x) = 1$ (resp. $\mu_{\tilde{A}}(x\alpha u) = \mu_{\tilde{A}}(x) = 1$)
$\nu_{\tilde{A}}(u\alpha(x+v) - u\alpha v) \leq \nu_{\tilde{A}}(x) = 0$ (resp. $\nu_{\tilde{A}}(x\alpha u) = \nu_{\tilde{A}}(x) = 0$)

So $(u\alpha(x+v) - u\alpha v) \in A$.
Hence A is a left (resp.right) ideal of M.

Theorem 3.4. Let A be an intuitionistic fuzzy left (resp.right) ideal of M and $t \in [0,1]$, then

I. $U(\mu_A ; t)$ is either empty or an ideal of M.
II. $L(\nu_A ; t)$ is either empty or an ideal of M.

Proof. (i) Let $x, y \in U(\mu_A ; t)$.
Then $\mu_A(x-y) \geq \{ \mu_A(x) \wedge \mu_A(y) \} \geq t$,
Hence $x-y \in L(\nu_A ; t)$.

$\mu_A(y + x - y) \geq \mu_A(x) \geq t$

Hence $(y + x - y) \in U(\mu_A ; t)$.

Let $x \in M$, $\alpha \in \Gamma$ and $u, v \in U(\mu_A ; t)$.
Then $\mu_A(u\alpha(x+v) - u\alpha v) \geq \mu_A(x) \geq t$ and so $(u\alpha(x+v) - u\alpha v) \in U(\mu_A ; t)$.
Hence $U(\mu_A ; t)$ is an ideal of M.

III. Let $x, y \in L(\nu_A ; t)$.
Then $\nu_A(x - y) \leq \{\nu_A(x) \vee \nu_A(y)\} \leq t$.
Hence $x-y \in L(\nu_A ; t)$.
$\nu_A(y + x - y) \leq \nu_A(x) \leq t$.
Hence $(y + x - y) \in L(\nu_A ; t)$.
Let $x \in M$, $\alpha \in \Gamma$ and $u, v \in L(\nu_A ; t)$.
Then $\nu_A(u\alpha(x+v) - u\alpha v) \leq \nu_A(x) \leq t$ and so $(u\alpha(x+v) - u\alpha v) \in L(\nu_A ; t)$.
Hence $L(\nu_A ; t)$ is an ideal of M.

Theorem 3.5. Let I be the left (resp.right) ideal of M. If the intuitionistic fuzzy set $A= < \mu_A,\nu_A >$ in M is defined by

$$\mu_A(x) = \begin{cases} p & if \quad x \in I \\ s & Otherwise \end{cases} \quad \text{and } \nu_A(x) = \begin{cases} u & if \quad x \in I \\ v & Otherwise \end{cases}$$

for all $x \in M$ and $\alpha \in \Gamma$, where $0 \leq s < p$, $0 \leq v < u$ and $p + u \leq 1$, $s + v \leq 1$, then A is an intuitionistic fuzzy left (resp.right) ideal of M and $U(\mu_A ; p) = I = L(\nu_A ; u)$.

Proof . Let $x, y \in M$ and $\alpha \in \Gamma$.
If at least one of x and y does not belong to I, then
$\mu_A(x-y) \geq s = \{\mu_A(x) \wedge \mu_A(y)\}$,
$\nu_A(x-y) \leq v = \{\nu_A(x) \vee \nu_A(y)\}$.
If $x, y \in I$, then
$x - y \in I$ and so $\mu_A(x-y) = p = \{\mu_A(x) \wedge \mu_A(y)\}$ and
$\nu_A(x-y) = v = \{\nu_A(x) \vee \nu_A(y)\}$.
$\mu_A(y + x - y) \geq s = \mu_A(x)$,
$\nu_A(y + x - y) \leq v = \nu_A(x)$.
If $x, y \in I$, then
$(y + x - y) \in I$ and so $\mu_A(y + x - y) = p = \mu_A(x)$ and
$\nu_A(y + x - y) = v = \nu_A(x)$.
If $u, v \in I$, $x \in M$ and $\alpha \in \Gamma$, then $(u\alpha(x+v) - u\alpha v) \in I$,
$\mu_A(u\alpha(x+v) - u\alpha v) = p = \mu_A(x)$ and $\nu_A(u\alpha(x+v) - u\alpha v) = u = \nu_A(x)$.
(resp. $\mu_A(x\alpha u) = p = \mu_A(x)$ and $\nu_A(x\alpha x) = u = \nu_A(x)$)

If $y \notin I$, then $\mu_A(u\alpha(x+v) - u\alpha v) = s = \mu_A(x)$, $\nu_A(u\alpha(x+v) - u\alpha v) = v = \nu_A(x)$.
(resp., $\mu_A(x\alpha u) = s = \mu_A(x)$ and $\nu_A(x\alpha u) = v = \nu_A(x)$)
Therefore A is an intuitionistic fuzzy left (resp.right) ideal.

Definition3.6. Let f be a mapping from a Γ–near-ring M onto a Γ–near-ring N. Let A be an intuitionistic fuzzy ideal of M. Now A is said to be f–invariant if f(x) = f(y) implies $\mu_A(x) = \mu_A(y)$ and $\nu_A(x) = \nu_A(y)$.

Definition 3.7 [2]. A function f : M → N, where M and N are Γ–near-rings, is said to be a Γ – homomorphism if f(a +b) = f(a) + f(b), f(aαb) = f(a)αf(b), for all a, b ∈ M and α ∈ Γ.

Definition 3.8 Let f : X → Y be a mapping of a Γ–near-ring and A be an intuitionistic fuzzy set of Y. Then the map $f^{-1}(A)$ is the pre-image of A under f, if $\mu_{f^{-1}(A)}(x) = \mu_A(f(x))$ and $\nu_{f^{-1}(A)}(x) = \nu_A(f(x))$, for all x ∈ X.

Definition 3.9. Let f be a mapping from a set X to the set Y. If A = <μ_A, ν_A> and B = < μ_B, ν_B > are intuitionistic fuzzy subsets in X and Y respectively, then the image of A under f is the intuitionistic fuzzy set f(A) = < $\mu_{f(A)}$, $\nu_{f(A)}$ > defined by

$$\mu_{f^{-1}(A)}(x) = \begin{cases} \bigvee_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \varphi, \\ 0 & \text{Otherwise,} \end{cases}$$

$$\nu_{f^{-1}(A)}(x) = \begin{cases} \bigwedge_{x \in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \varphi, \\ 1 & \text{Otherwise,} \end{cases}$$

for all y ∈Y.

(b) the pre image of A under f is the intuitionistic fuzzy set $f^{-1}(B)$ = < $\mu_{f^{-1}(B)}$, $\nu_{f^{-1}(B)}$ > defined by

$$\mu_{f^{-1}(B)}(x) = \begin{cases} \bigvee_{y \in f^{-1}(x)} \mu_B(y) & \text{if } f^{-1}(x) \neq \varphi, \\ 0 & \text{Otherwise,} \end{cases}$$

$$\nu_{f^{-1}(B)}(x) = \begin{cases} \bigwedge_{y \in f^{-1}(x)} \nu_A(y) & \text{if } f^{-1}(x) \neq \varphi, \\ 1 & \text{Otherwise,} \end{cases}$$

for all x ∈X, where $\mu_{f^{-1}(B)}(x) = \mu_B(f(x))$ and $\nu_{f^{-1}(B)}(x) = \nu_B(f(x))$.

Theorem 3.10. Let M and N be two Γ–near-rings and θ : M → N be a Γ– epimorphism and let B = < μ_B, ν_B > be an intuitionistic fuzzy set of N. If $\theta^{-1}(B)$ = < $\mu_{\theta^{-1}(B)}$, $\nu_{\theta^{-1}(B)}$ > is an intuitionistic fuzzy left (resp.right) ideal of M, then B is an intuitionistic fuzzy left (resp.right) ideal of N.

Proof. Let x, y, u, v ∈ N and α∈Γ, then there exists a, b, c, d ∈ M such that θ(a) = x, θ(b) = y, θ(c) = u, θ(d) = v.

It follows that $\mu_B(x-y) = \mu_B(\theta(a)-\theta(b)) = \mu_B(\theta(a-b))$

$= \mu_{\theta^{-1}(B)}(a-b) \geq \{ \mu_{\theta^{-1}(B)}(a) \wedge \mu_{\theta^{-1}(B)}(b)\}$

$= \{\mu_B(\theta(a)) \wedge \mu_B(\theta(b))\}$

$= \{\mu_B(x) \wedge \mu_B(y)\}$
$\nu_B(x-y) = \nu_B(\theta(a)-\theta(b)) = \nu_B(\theta(a-b))$
$= \nu_{\theta^{-1}(B)}(a-b) \leq \{\nu_{\theta^{-1}(B)}(a) \vee \nu_{\theta^{-1}(B)}(b)\}$
$= \{\nu_B(\theta(a)) \vee \nu_B(\theta(b))\}$
$= \{\nu_B(x) \vee \nu_B(y)\}$.
$\mu_B(y + x - y) = \mu_B(\theta(b) + \theta(a)-\theta(b)) = \mu_B(\theta(b + a-b))$
$= \mu_{\theta^{-1}(B)}(b + a-b) \geq \mu_{\theta^{-1}(B)}(a)$
$= \mu_B(\theta(a))$
$= \mu_B(x)$
$\nu_B(y + x - y) = \nu_B(\theta(b) + \theta(a)-\theta(b)) = \nu_B(\theta(b + a-b))$
$= \nu_{\theta^{-1}(B)}(b + a-b) \leq \nu_{\theta^{-1}(B)}(a)$
$= \nu_B(\theta(a))$
$= \nu_B(x)$.
Also
$\mu_B(u\alpha(x +v) -u\alpha v) = \mu_B(\theta(c)\alpha(\theta(a)+\theta(d))-\theta(c)\alpha\theta(d)) = \mu_B(\theta(c\alpha(a+d)-c\alpha d))$
$= \mu_{\theta^{-1}(B)}(c\alpha(a+d)-c\alpha d) \geq \mu_{\theta^{-1}(B)}(a) = \mu_B(\theta(a)) = \mu_B(x)$
$\nu_B(u\alpha(x +v) -u\alpha v) = \nu_B(\theta(c)\alpha(\theta(a)+\theta(d))-\theta(c)\alpha\theta(d)) = \nu_B(\theta(c\alpha(a+d)-c\alpha d))$
$= \nu_{\theta^{-1}(B)}(c\alpha(a+d)-c\alpha d) \leq \nu_{\theta^{-1}(B)}(a) = \nu_B(\theta(a)) = \mu_B(x)$

Similarly, $\mu_B(x\alpha u) \geq \mu_B(x)$ and $\nu_B(x\alpha u) \leq \nu_B(x)$.
Hence B is an intuitionistic fuzzy left (resp. right) ideal of N.

Theorem 3.11. An intuitionistic fuzzy set $A= < \mu_A, \nu_A >$ in a Γ–near-ring M is an intuitionistic fuzzy left (resp. right) ideal if and only if $A_{<t,s>} =\{ x\in M \mid \mu_A(x) \geq t, \nu_A(x) \leq s\}$ is a left (resp.right) ideal of M for $\mu_A(0) \geq t$, $\nu_A(0) \leq s$.

Proof.(⇒) Suppose that $A = < \mu_A, \nu_A >$ is an intuitionistic fuzzy left (resp.right) ideal of M and let $\mu_A(0) \geq t$, $\nu_A(0) \leq s$. Let $x, y, u, v \in A_{<t,s>}$ and $\alpha\in \Gamma$.

Then $\mu_A(x) \geq t$, $\nu_A(x) \leq s$ and $\mu_A(y) \geq t$, $\nu_A(y) \leq s$.
Hence $\mu_A(x-y) \geq \{\mu_A(x) \wedge \mu_A(y)\} \geq t$,
$\nu_A(x-y) \leq \{\nu_A(x) \vee \nu_A(y)\} \leq s$.
$\mu_A(y + x - y) \geq \mu_A(x) \geq t$,
$\nu_A(y + x - y) \leq \nu_A(x) \leq s$.
$\mu_A(u\alpha(x +v) -u\alpha v) \geq \mu_A(x) \geq t$ and $\nu_A(u\alpha(x +v) -u\alpha v) \leq \nu_A(x) \leq s$
(resp. $\mu_A(x\alpha u) \geq \mu_A(x) \geq t$ and $\nu_A(x\alpha u) \leq \nu_A(x) \leq s$).

Therefore $x-y \in A_{<t,s>}$, $(y + x - y)\in A_{<t,s>}$ and $(u\alpha(x +v) -u\alpha v) \in A_{<t,s>}$ for all x, $y \in A_{<t,s>}$ and $\alpha\in\Gamma$.

So $A_{<t,s>}$ is a left (resp. right) ideal of M.

(⇐) Suppose that $A_{<t,s>}$ is a left (resp. right) ideal of M for $\mu_A(0) \geq t$ and $\nu_A(0) \leq s$.

Let $x, y \in M$ be such that $\mu_A(x) = t_1$, $\nu_A(x) = s_1$, $\mu_A(y) = t_2$ and $\nu_A(y) = s_2$.

Then $x \in A_{<t_1, s_1>}$ and $y \in A_{<t_2, s_2>}$.
We may assume that $t_2 \leq t_1$ and $s_2 \geq s_1$ without loss of generality.
It follows that $A_{<t_2, s_2>} \subseteq A_{<t_1, s_1>}$ so that $x, y \in A_{<t_1, s_1>}$.
Since $A_{<t_1, s_1>}$ is an ideal of M, we have $x-y \in A_{<t_1, s_1>}$, $(y + x - y) \in A_{<t_1, s_1>}$ and
$(u\alpha(x+v) - u\alpha v) \in A_{<t_1, s_1>}$ for all $\alpha \in \Gamma$.
$\mu_A(x-y) \geq t_1 \geq t_2 = \{\mu_A(x) \wedge \mu_A(y)\}$,
$\nu_A(x-y) \leq s_1 \leq s_2 = \{\nu_A(x) \vee \nu_A(y)\}$.
$\mu_A(y + x - y) \geq t_1 \geq t_2 = \mu_A(x)$,
$\nu_A(y + x - y) \leq s_1 \leq s_2 = \nu_A(x)$.
$\mu_A(u\alpha(x+v) - u\alpha v) \geq t_1 \geq t_2 = \mu_A(x)$ and
$\nu_A(u\alpha(x+v) - u\alpha v) \leq s_1 \leq s_2 = \nu_A(x)$.

Therefore A is an intuitionistic fuzzy left (resp.right) ideal of M.

Theorem 3.12. If the IFS $A = < \mu_A, \nu_A >$ is an intuitionistic fuzzy left (resp.right) ideal of a Γ–near-ring M, then the sets $M\mu_A = \{x \in M / \mu_A(x) = \mu_A(0)\}$ and $M\nu_A = \{x \in M / \nu_A(x) = \nu_A(0)\}$ are left (resp.right) ideals.

Proof. Let $x, y, u, v \in M\mu_A$ and $\alpha \in \Gamma$.
Then $\mu_A(x) = \mu_A(0)$, $\mu_A(y) = \mu_A(0)$.
Since A is an intuitionistic fuzzy left (resp.right) ideal of a Γ–near-ring M, we get
$\mu_A(x - y) \geq \{\mu_A(x) \wedge \mu_A(y)\} = \mu_A(0)$.

But $\mu_A(0) \geq \mu_A(x-y)$. So $x-y \in M\mu_A$.
$\mu_A(y + x - y) \geq \mu_A(x) = \mu_A(0)$.

But $\mu_A(0) \geq \mu_A(y + x - y)$. So $(y + x - y) \in M\mu_A$.
$\mu_A(u\alpha(x+v) - u\alpha v) \geq \mu_A(x) = \mu_A(0)$ (resp. $\mu_A(x\alpha u) \geq \mu_A(x) = \mu_A(0)$).

Hence $(u\alpha(x+v) - u\alpha v) \in M\mu_A$.
Therefore $M\mu_A$ is a left (resp.right) ideal of M.
Similarly, let $x, y, u, v \in M\nu_A$ and $\alpha \in \Gamma$. Then $\nu_A(x) = \nu_A(0)$, $\nu_A(y) = \nu_A(0)$.
Since A is an intuitionistic fuzzy left (resp.right) ideal of a Γ–near-ring M,
$\nu_A(x - y) \leq \{\nu_A(x) \vee \nu_A(y)\} = \nu_A(0)$.
But $\nu_A(0) \leq \nu_A(x-y)$. So $x-y \in M\nu_A$.
$\nu_A(y + x - y) \leq \nu_A(x) = \nu_A(0)$.
But $\nu_A(0) \leq \nu_A(y + x - y)$. So $(y + x - y) \in M\nu_A$.
$\nu_A(u\alpha(x+v) - u\alpha v) \leq \nu_A(x) = \nu_A(0)$ (resp. $\nu_A(x\alpha u) \leq \nu_A(x) = \nu_A(0)$).
Hence $(u\alpha(x+v) - u\alpha v) \in M\nu_A$.
Therefore $M\nu_A$ is a left (resp.right) ideal of M.

Definition 3.13. A Γ–near-ring M is said to be regular if for each $a \in M$ there exists an $x \in M$ and $\alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$.

Definition 3.14. Let $A = <\mu_A, \nu_A>$ and $B = <\mu_B, \nu_B>$ be two intuitionistic fuzzy subsets of a Γ-near-ring M. The product AΓB is defined by

$$\mu_{A\Gamma B}(x) = \begin{cases} \bigvee_{x=(u\gamma(v+w)-u\gamma w)} \left(\mu_A(u) \wedge \mu_B(v)\right) & \text{if } x=(u\gamma(v+w)-u\gamma w), u,v,w \in M, \ \gamma \in \Gamma \\ 0 & \text{otherwise,} \end{cases}$$

$$\nu_{A\Gamma B}(x) = \begin{cases} \bigwedge_{x=(u\gamma(v+w)-u\gamma w)} \left(\nu_A(u) \vee \nu_B(v)\right) & \text{if } x=(u\gamma(v+w)-u\gamma w), u,v,w \in M, \ \gamma \in \Gamma \\ 1 & \text{otherwise.} \end{cases}$$

Theorem 3.15. If $A = <\mu_A, \nu_A>$ and $B = <\mu_B, \nu_B>$ are two intuitionistic fuzzy left (resp. right) ideals of M, then A∩B is an intuitionistic fuzzy left (resp. right) ideal of M. If A is an intuitionistic fuzzy right ideal and B is an intuitionistic fuzzy left ideal, then $A\Gamma B \subseteq A \cap B$.

Proof. Suppose A and B are intuitionistic fuzzy ideals of M and let $x, y, z, z' \in M$ and $\alpha \in \Gamma$.

Then

$\mu_{A\cap B}(x-y) = \mu_A(x-y) \wedge \mu_B(x-y)$
$\geq [\mu_A(x) \wedge \mu_A(y)] \wedge [\mu_B(x) \wedge \mu_B(y)]$
$= [\mu_A(x) \wedge \mu_B(x)] \wedge [\mu_A(y) \wedge \mu_B(y)]$
$= \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$,
$\nu_{A\cap B}(x-y) = \nu_A(x-y) \vee \nu_B(x-y)$
$\leq [\nu_A(x) \vee \nu_A(y)] \vee [\nu_B(x) \vee \nu_B(y)]$
$= [\nu_A(x) \vee \nu_B(x)] \vee [\nu_A(y) \vee \nu_B(y)]$
$= \nu_{A\cap B}(x) \vee \nu_{A\cap B}(y)$.
$\mu_{A\cap B}(y + x-y) = \mu_A(y + x-y) \wedge \mu_B(y + x-y)$
$\geq [\mu_A(x)] \wedge [\mu_B(x)]$
$= \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$,
$\nu_{A\cap B}(y + x-y) = \nu_A(y + x-y) \vee \nu_B(y + x-y)$
$\leq [\nu_A(x)] \vee [\nu_B(x)]$
$= \nu_{A\cap B}(x) \vee \nu_{A\cap B}(y)$.

Since A and B are intuitionistic fuzzy ideals of M, we have
$\mu_A(x\alpha(y+z)-x\alpha z) \geq \mu_A(x)$, $\nu_A(x\alpha(y+z)-x\alpha z) \leq \nu_A(x)$ and $\mu_B(y\alpha x) \geq \mu_B(x)$, $\nu_B(y\alpha x) \leq \nu_B(x)$.

Now $\mu_{A\cap B}(x\alpha(y+z)-x\alpha z) = \mu_A(x\alpha(y+z)-x\alpha z) \wedge \mu_B(x\alpha(y+z)-x\alpha z)$
$\geq \mu_A(x) \wedge \mu_B(x) = \mu_{A\cap B}(x)$(resp. $\mu_{A\cap B}(y\alpha x) \geq \mu_{A\cap B}(x)$),
$\nu_{A\cap B}(x\alpha(y+z)-x\alpha z) = \nu_A(x\alpha(y+z)-x\alpha z) \vee \nu_B(x\alpha(y+z)-x\alpha z)$
$\geq \nu_A(x) \vee \nu_B(x) = \nu_{A\cap B}(x)$(resp. $\nu_{A\cap B}(y\alpha x) \leq \nu_{A\cap B}(x)$).

Hence A∩B is an intuitionistic fuzzy left ideal of M.

To prove the second part if $\mu_{A\Gamma B}(x) = 0$ and$\nu_{A\Gamma B}(x) = 1$, there is nothing to show.
From the definition of $A\Gamma B$, $\mu_A(x) = \mu_A(y\alpha(z+z')- y\alpha z') \geq \mu_A(z)$,
$\nu_A(x) = \nu_A(y\alpha(z+z')- y\alpha z') \leq \nu_A(z)$.

Since A is an intuitionistic fuzzy right ideal and B is an intuitionistic fuzzy left ideal, we have
$\mu_A(x) = \mu_A(z\alpha y) \geq \mu_A(z)$, $\nu_A(x) = \nu_A(z\alpha y) \leq \nu_A(z)$,
$\mu_B(x) = \mu_B(z\alpha y) \geq \mu_B(z)$, $\nu_B(x) = \nu_B(z\alpha y) \leq \nu_B(z)$.
Hence by Definition 3.5,

$$\mu_{A\Gamma B}(x) = \bigvee_{x=y\alpha z} \{\mu_A(y) \wedge \mu_B(z)\}$$
$$\leq \mu_A(x) \wedge \mu_B(x)$$
$$= \mu_{A\cap B}(x)$$

and

$$\nu_{A\Gamma B}(x) = \bigwedge_{x=y\alpha z} \{\nu_A(y) \vee \nu_B(z)\}$$
$$\geq \nu_A(x) \vee \nu_B(x)$$

$= \nu_{A\cap B}(x)$ which means that $A\Gamma B \subseteq A\cap B$.

Theorem 3.16. A Γ−near-ring M is regular if and only if for each intuitionistic fuzzy right ideal A and each intuitionistic fuzzy left ideal B of M , $A\Gamma B = A\cap B$.

Proof. (⇒) Suppose R is regular. $A\Gamma B \subseteq A\cap B$. Thus it is sufficient to show that $A\cap B \subseteq A\Gamma B$. Let $a \in M$ and $\alpha, \beta \in \Gamma$. Then, by hypothesis, there exists an $x \in M$ such that $a = a\alpha x\beta a$.
Thus
$\mu_A(a) = \mu_A(a\alpha x\beta a) \geq \mu_A(a\alpha x) \geq \mu_A(a)$,
$\nu_A(a) = \nu_A(a\alpha x\beta a) \leq \nu_A(a\alpha x) \leq \nu_A(a)$.

So $\mu_A(a\alpha x) = \mu_A(a)$ and $\nu_A(a\alpha x) = \nu_A(a)$.
On the other hand,
$\mu_{A\Gamma B}(a) = \vee_{a= a\alpha x\beta a} [\mu_A(a\alpha x) \wedge \mu_B(a)] \geq [\mu_A(a) \wedge \mu_B(a)] = \mu_{A\cap B}(a)$ and
$\nu_{A\Gamma B}(a) = \wedge_{a= a\alpha x\beta a} [\nu_A(a\alpha x) \vee \nu_B(a)] \leq [\nu_A(a) \vee \nu_B(a)] = \nu_{A\cap B}(a)$.
Thus $A\cap B \subseteq A\Gamma B$. Hence $A\Gamma B = A\cap B$.

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