International Journal of Computational Science and Mathematics.
ISSN 0974-3189 Volume 3, Number 2 (2011), pp. 245-254

http://www.irphouse.com

Characterization of Two Domination Number and Chromatic Number of a Graph

G. Mahadevan, A. Selvam Avadyappan and A. Mydeen bibi

[1]*Department of Mathematics,*
Anna University of Technology, Tirunelveli-627002, India.
E-mail: gmaha2003@yahoo.co.in
[2]*Department of Mathematics, V.H.N.S.N. College, Virudhunagar, India.*
E-mail: avadayappan@yahoo.co.in
[3]*Research Scholar, Mother Teresa Women's University, Kodaikanal, India.*
E-mail: amydeen2006@yahoo.co.in

Abstract

A Subset S of V is called a dominating set in G if every vertex in V-S is adjacent to at least one vertex in S. A Dominating set is said to be two dominating set if every vertex in V-S is adjacent to at least two vertices in S. The minimum cardinality taken over all, the minimal two dominating set is called two domination number and is denoted by $\gamma_2(G)$. The minimum number of colors required to colour all the vertices such that adjacent vertices do not receive the same colour is the chromatic number $\chi(G)$. In this paper, we characterize the classes of graphs whose sum of two domination number and chromatic number is equals to 2n-5 and 2n – 6.

Keywords: Two Domination Number, Chromatic Number.

AMS Subject Classification: 05C.

Introduction

Let G= (V, E) be a simple undirected graph. The degree of any vertex u in G is the number of edges incident with u and is denoted by d(u). The minimum and maximum degree of a vertex is denoted by $\delta(G)$ and $\Delta(G)$ respectively; P_n denotes the path on n vertices. The vertex connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected graph. A colouring of a graph is an assignment of colours to its vertices so that two adjacent vertices have the same color. An n-colouring of a graph G uses n colours. The Chromatic Number χ is defined to be the minimum n for which G has an n-colouring. If $\chi(G) = k$ but $\chi(G) < k$, for every proper subgraph H of G, then G is k-critical.

A subset S of V is called a dominating set in G if every vertex in V-S is adjacent to atleast one vertex in S. The minimum cardinality taken over all dominating sets in G is called the domination number of G and is denoted by γ. A dominating set is said to be two dominating set if every vertex in V-S is adjacent to atleast two vertices in S. The minimum cardinality taken over all the minimal two dominating set is called two domination number and is denoted by $\gamma_2(G)$.

Several authors have studied the problem of obtaining an upper bound for the sum of a domination parameter and a graph theoretic parameter and characterized the corresponding extremal graphs. In [10], Paulraj Joseph J and Arumugam S proved that $\gamma+k \leq p$. In [11], Paulraj Joseph J and Arumugam S proved that $\gamma_c+\chi=p+1$. They also characterized the class of grahs for which the upper bound is attained. They also proved similar results for γ and γ_t. In [7], Paulraj Joseph J and Mahadevan G, proved that $\gamma_{cc} + \chi \leq 2n-1$ and characterized the corresponding extremal graphs. In [12], Paulraj Joseph J and Mahadevan G proved that $\gamma_{pr}+\chi \leq 2n-1$ and characterized the corresponding extremal graphs. In [8], Paulraj Joseph J and Mahadevan G introduced the concept of complementary perfect domination number γ_{cp} and proved that $\gamma_{cp}+\chi \leq 2n-2$, and characterized the corresponding extremal graphs. They also obtained the similar results for the induced complementary perfect domination number and chromatic number of a graph. In this paper, we obtain sharp upper bound for the sum of the two domination number and chromatic number and characterize the corresponding extremal graphs. Terms not defined here, are used in the sense of Hanary[1].

Notations: $K_n(P_m)$ denotes the graph obtained from K_n by attaching the end vertex of P_m to any one of the vertices of K_n. $K_n(m_1,m_2,m_3,\ldots..m_k)$ denotes the graph obtained from K_n by attaching m_1 edges to the vertex u_i of K_n, m_2 edges to the vertex u_j for $i \neq j$ of K_n ………, m_k edges to all the distinct vertices of K_n.

Previous Results

Theorem 1.1: For any connected graph G, $\gamma_2(G) \leq n$

Theorem 1.2: For any connected graph G, $\chi(G) \leq \Delta(G) + 1$

Theorem 1.3: For any connected graph G, $\gamma_2(G) + \chi(G) \leq 2n$ and the equality holds if and if only $G \cong K_2$.

Main Results

Theorem 2.1: For any connected graph G, $\gamma_2(G)+\chi(G)=2n-5$ if and only if $G \cong K_3(4,0,0)$, $K_3(3,1,0)$, $K_3(2,2,0)$, $S(K_{1,5})$, $K_4(1,1,1,0)$, $K_4(3,0,0,0)$, $K_4(2,1,0,0)$, $K_3(P_4)$, $K_3(P_3,P_2,0)$, $K_4(P_3)$, $K_5(2,0,0,0,0)$, $K_5(2,2,0,0,0)$, $K_6(P_2)$, K_7 or one of the following graphs in the figure 2.1.

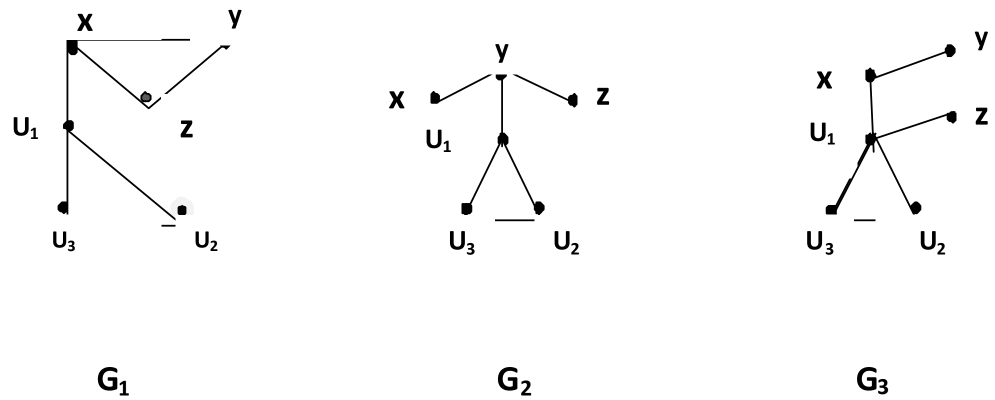


Figure 2.1

Proof: If G is anyone of the graph given in the figure, then clearly $\gamma_2(G) + \chi(G)$=2n-5. Conversely assume that $\gamma_2(G) + \chi(G)$=2n-5. This is possible only ifγ_2=n,χ =n-5(or) γ_2 =n-1, χ = n-4(or)γ_2 =n-2, χ = n-3(or) γ_2 =n-3, χ =n-2(or)γ_2 =n-4, χ =n-1(or) γ_2 =n-5, χ =n.

Case (i) Let γ_2=n andχ=n-5, since χ=n-5, G contains a clique K on n-5 vertices. Let S={x_1,x_2,x_3,x_4,x_5}$\in$V-S. Then <S> =K_5, $\overline{K_5}$,P_5,$K_4\cup K_1$,$P_3\cup K_2$,$K_{1,4}$,$P_2\cup K_3$ $K_{2,3}$, $K_3\cup K_2$. In all the above cases, it can be verified that no new graph exists.

Case (ii) Let γ_2=n-1and χ=n-4 since χ=n-4, G contains a clique K on n-4 vertices. Let S={x_1,x_2,x_3,x_4}. Then< S> =K_4, $\overline{K_4}$,P_4,$K_3\cup K_1$,$K_{1,3}$,$K_2\cup K_2$,$P_3\cup K_1$.

If <S>=K_4, then no graph exists.

Subcase(a) Let <S>= $\overline{K_4}$, since G is connected. One of the vertices of K_{n-4} say u_i is adjacent to all the vertices of S (or) three vertices of S are adjacent to the vertex u_i and the fourth one is adjacent to u_j for $i\neq j$(or)two vertices of S are adjacent to the vertex u_i and the remaining vertices of S are adjacent to u_j (or) two vertices of S are adjacent to the vertex u_i and in the remaining vertices one is adjacent to u_j and another one is adjacent to u_k for $i\neq j\neq k$(or)all the vertices of S are adjacent to the distinct vertices of K_{n-4}. Then in all the cases,{x_1,x_2,x_3,x_4,u_i,u_j}for $i\neq j$ is a γ_2 set. So that γ_2 = 6, since γ_2 = n-1. So that n=7. Since χ = n-4= 3. Hence K = K_3. Let u_1,u_2,u_3 be the vertices of K_3. If all the vertices of S are adjacent to u_1 then γ_2=6 and $d(x_1)$ =$d(x_2)$ = $d(x_3)$ = $d(x_4)$ =1. Hence G $\cong$ G_1. If three vertices of S are adjacent to u_1 and the fourth one is adjacent to u_2 then γ_2 =6 and $d(x_1)$ =$d(x_2)$ =$d(x_3)$ =$d(x_4)$ =1. Hence G $\cong$ K_3 (3, 1, 0). If two vertices of S are adjacent to u_1 and the remaining two vertices are adjacent to u_2, then γ_2 = 6 and $d(x_1)$ =$d(x_2)$ =$d(x_3)$ =$d(x_4)$ =1. Hence G $\cong$ K_3 (2, 2, 0).

Subcase(b) Let <S>=P_4=(x_1,x_2,x_3,x_4), since G is connected. There exists a vertex say u_i in K_{n-4} is adjacent to x_1(or equivalently x_4) (or) x_2(or equivalently x_3). Let u_i be adjacent to x_1. Then $\{x_2,x_4,u_i,u_j\}$ for $i\neq j$ is a γ_2 set. So that γ_2 =4, since γ_2=n-1 implies that n=5. Since χ= n-4 =1 which is a contradiction. Since G is totally disconnected. Hence no graph exists. Let u_i be adjacent to x_2. Then $\{x_1,x_2,x_4,u_i,u_j\}$ for $i\neq j$ is a γ_2 set. So that γ_2 = 5, since γ_2 = n-1 implies that n = 6, since χ = n-4 = 2. Hence K = K_2 = uv. If u is adjacent to x_1 then γ_2 = 4 which is a contradiction. Hence no graph exists.

Subcase(c) Let <S>=$K_{1,\ 3}$. Let the vertex x_1 be adjacent to x_2,x_3,x_4. Since G is connected, there exists a vertex u_i inK_{n-4} which is adjacent to x_1 or any one of $\{x_2,x_3,x_4\}$. let u_i be adjacent to $x_{1,}$ then $\{x_2,x_3,x_4,u_i,u_j\}$ for $i\neq j$ is a γ_2 set so that γ_2 = 5, since γ_2 = n-1 implies that n=6. Since χ = n-4 =2. Hence K=K_2=uv. If u is adjacent to $x_{1,}$ then γ_2 =5 and $d(x_1)$ =$d(x_2)$ =$d(x_3)$ =$d(x_4)$ =1. Hence G $\cong$ S ($K_{1,5}$). If u is adjacent to x_4 then γ_2 =4, which is a contradiction. Hence no graph exists.

For all the remaining cases, no new graph exists.

Case (iii) Let γ_2= n -2and χ = n-3, since χ =n-3, G contains a clique K on (n-3) vertices. Let S =$\{x,y,z\}\in$ V-S. <S>=K_3, $\overline{K}_3$,P_3,$K_2\cup K_1$,$P_2\cup K_1$.

Subcase(a) Let <S> =K_3, since G is connected, x is adjacent to some u_i in K_{n-3}. Then $\{x,y,u_i,u_j\}$ for $i\neq j$ is a γ_2 set, so that γ_2 =4. Now γ_2 = n-2, since n=6. Since χ=n-3=3. Hence K = K_3. Let u_1,u_2,u_3 be the vertices K_3. Without loss of generality, u_1 is adjacent to x, then γ_2 = 4. If d(x) =d(y) = d(z) =2. Then G$\cong$ G_1.

Subcase(b) Let <S>= $\overline{K}_3$, since G is connected, one of the vertices of K_{n-3} say u_i is adjacent to all the vertices of S (or) two vertices of S (or) one vertex of S. If u_i for some i is adjacent to all the vertices of S then $\{x,y,z,u_i,u_j\}$ for $i\neq j$ is a γ_2 set. Now γ_2 =5, since γ_2 =n-2, so that n=7 since χ =n-3=4. Hence K=K_4. Let u_1,u_2,u_3,u_4 be the vertices of K_4. Without loss of generality, u_1 is adjacent to all the vertices of S and d(x) =d(y) = d(z) =1. Since γ_2 =5, then G $\cong$ K_4 (3, 0, 0, 0). If u_1 is adjacent to x and y and u_2 is adjacent to z, since γ_2 =5, then G $\cong$ K_4 (2, 1, 0, 0). If u_1 is adjacent to x and u_2 is adjacent is to y and u_3 is adjacent to z since γ_2=5, d(x) =d(y) =d(z) =1. Then G$\cong$$K_4$(1,1,1,0).

Subcase(c) Let <S>=P_3 =(x,y,z). Since G is connected (or equivalently z) is adjacent to u_i for some i in K_{n-3}. Then $\{x,z,u_i,u_j\}$for $i\neq j$ is a γ_2 set or if y is adjacent to u_i in K_{n-3} then the same γ_2 set arise. So that n=6. Hence K=K_3. Let u_1,u_2,u_3 be the vertices of K_3. If u_1 is adjacent to x, then γ_2=4. Hence G$\cong$$K_3$ (P_3). If u_1 is adjacent to y, then γ_2=4. Hence G$\cong$$G_2$.

Subcase(d) Let<S>=$K_2\cup K_1$. Let xy be the edge in $K_2\cup K_1$, since G is connected. There exists a u_i in K_{n-3} is adjacent to x. If z is adjacent to same u_i or z is adjacent to u_j for $i\neq j$ in K_{n-3}. Then $\{y,z,u_i,u_j\}$ for $i\neq j$ is a γ_2 set. So that n=6. Hence K = K_3. Let u_1,u_2,u_3 be the vertices of K_3. Let u_1 be adjacent to x and z then γ_2 = 4. Hence G $\cong$ G_3. Let u_1 be adjacent to x and u_2 be adjacent to z then γ_2 = 4. Hence G$\cong$$K_3$ (2, 1, 0).

Case (iv) Let γ_2 = n-3and χ = n-2, since χ = n-2, G contains a clique K on n-2 vertices. Let S=$\{x,y\}\in$V-S. <S>=K_2 or $\overline{K}_2$.

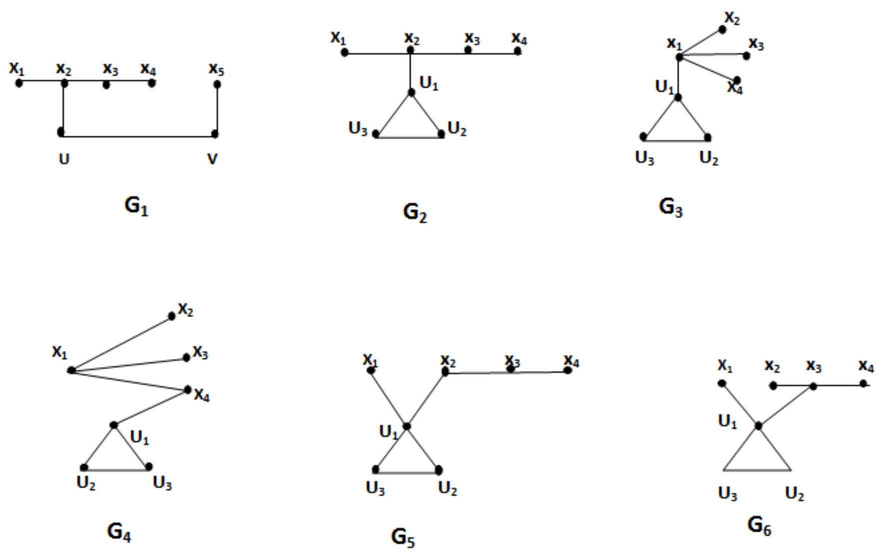
Subcase(a) Let <S>=K_2, since G is connected. There exists a vertex u_i in K_{n-2} is adjacent to x. Then $\{y,u_i,u_j\}$ is a γ_2 set. So that n=6. Hence K=K_4. Let u_1,u_2,u_3,u_4 be the vertices of K_4. Let u_1 be adjacent to x, if d(x) =2 and d(y) =1, then G ≅ $K_4(P_3)$.

Subcase(b) Let <S> = $\overline{K}_2$, since G is connected. There exists a vertex u_i in K_{n-2} is adjacent to x and y (or) x is adjacent to u_i and y is adjacent to u_j. In both the cases,$\{x,y,u_i,u_j\}$ is a γ_2 set . So that n=7, hence K=K_5. Let u_1,u_2,u_3,u_4,u_5 be the vertices of K_5. Let x and y be adjacent to u_1 then γ_2 =4. If d(x)=d(y)=1, then G≅K_5(2,0,0,0,0). Let u_1 be adjacent to x and u_2 be adjacent to y, then γ_2=4. If d(x)=d(y)=1, then G≅K_5(1,1,0,0,0).

Case (v) Let γ_2 = n-4and χ = n-1, since χ = n-1, G contains a clique K on n-1 vertices or does not contain a clique on n-1 vertices. There exists a vertex u_i in K_{n-1} is adjacent to x. Then$\{ x,u_i,u_j\}$ for i≠j is a γ_2 set. So that n = 7. Hence K=K_6. Let u_1,u_2,u_3,u_4,u_5 ,u_6 be the vertices of K_6. Let u_1 is adjacent to x, so that γ_2 =3. If d(x)=1, then G ≅ $K_6(P_2)$.

Case (vi) Let γ_2 = n-5 and χ = n, since χ=n. Then G= K_n , but for K_n, γ_2 =2 implies that n=7. Hence G ≅ K_7.

Theorem 2.2: For any connected G, $\gamma_2(G) + \chi(G)$ = 2n-6 if and if only G≅K_3(5,0,0), S($K_{1,6}$), K_4(4,0,0,0), K_4(3,1,0,0), K_4(2,2,0,0), P_6, K_3(1,3,0), K_5(3,0,0,0,0), K_5(2,1,0,0,0), K_5(1,1,1,0,0), $K_4(P_4)$, $K_5(P_3)$, K_6(2,0,0,0,0,0), K_6(1,1,0,0,0,0), K_7(1,0,0,0,0,0,0), K_8 or any one of the following graphs in the figure2.2.



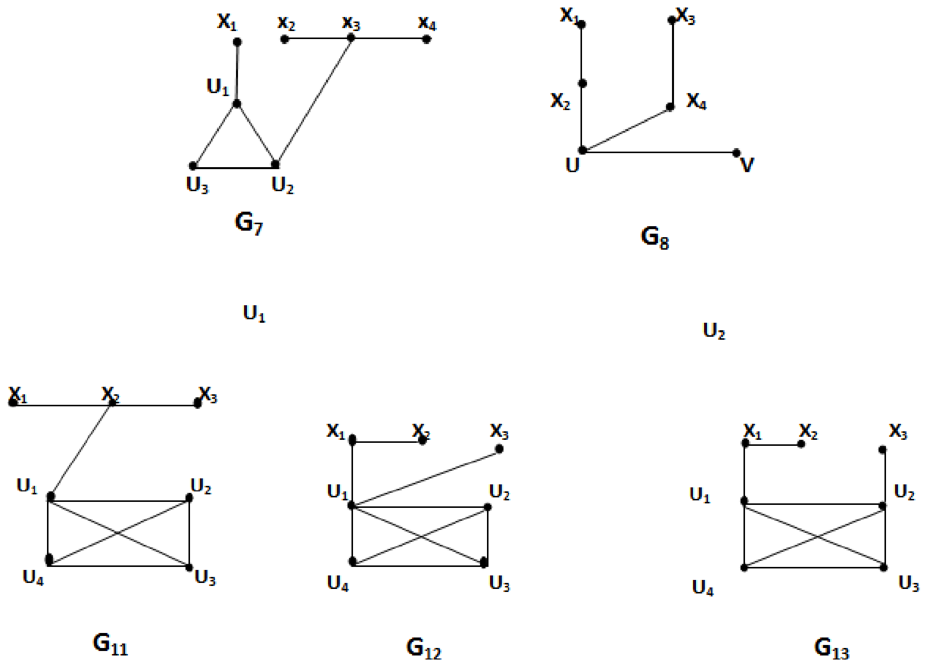


Figure 2.2

Proof: If G is anyone of the graph given in the figure, then clearly γ_2 (G) + χ(G) = 2n-6, conversely assume that γ_2(G) + χ(G) = 2n-6. This is possible only if γ_2(G) = n, χ (G) = n-6(or)γ_2 (G) = n-1, χ(G) = n-5(or)γ_2 (G) = n-2, χ (G)= n-4 (or) γ_2(G) = n-3, χ(G) = n-3(or)γ_2 (G) = n-4, χ (G) = n-2(or)γ_2(G) = n-5, χ (G) = n-1(or) γ_2 (G) = n-6, χ (G) = n.

Case (i) Let γ_2 (G)=n, χ (G)= n-6, since χ=n-6, G contains a clique K on n-6 vertices. Let S = $\{x_1,x_2,x_3,x_4,x_5,x_6\}\in$V-S. Then <S> = K_6, $\overline{K}_6$,P_6, $K_3 \cup K_3$,$K_2 \cup K_4$,$P_3 \cup K_3$,$P_2 \cup K_4$,$K_{1,5}$,$K_{3,3}$,$K_{2,4}$

If <S> = K_6, then no graph exists.

Subcase(a) Let <S> = $\overline{K}_6$. Since G is connected, one of the vertices of K_{n-6} say u_i is adjacent to all the vertices of S(or)five vertices of S(or)four vertices of S (or)three vertices of S(or)two vertices of S (or)one vertex of S. In all the cases, $\{x_1,x_2,x_3,x_4,x_5,x_6,u_i,u_j\}$ for i $\neq$ j forms a γ_2 set. So that γ_2 =8. Hence n = 8. So that K= K_2 = uv. In all the cases, no new graph exists.

For all the remaining cases, no new graph exists.

Case (ii) Let γ_2 (G) =n-1, χ(G) =n-5. Since χ= n–5, G contains a clique K on n – 5 vertices or does not contain a clique K on n-5 vertices. Let S=$\{x_1, x_2, x_3, x_4, x_5\}\in$ V-S . Then< S > = K_5 , $\overline{K}_5$, P_5 , $P_3 \cup K_2$,$K_3 \cup K_2$, $K_4 \cup K_1$,$P_4 \cup K_1$, $K_{1,4}$, $K_{2,3}$.

If<S> = K_5, then no graph exists.

Subcase(a) Let<S> = $\overline{K}_5$. Since G is connected. There exists a vertex u_i in K_{n-5} is a adjacent to all the vertices of S (or) four vertices of S (or) three vertices of S (or) two vertices on S (or) one vertex of S. Then in all the cases, { $x_1,x_2,x_3,x_4,x_5,u_i,u_j$ } for i $\neq$ j is a γ_2 set. So that $\gamma_2 = 7$. Hence n = 8. So that K = K_3. Let u_1,u_2,u_3, be the vertices of K_3. If u_1 is adjacent to all the vertices of S and if $d(x_1)=d(x_2)=d(x_3)=d(x_4)=d(x_5)=1$, then G≅$K_3(5,0,0)$. In all other cases, no new graph exists.

Subcase(b) Let<S> = $P_4 \cup K_1$. Since G is connected. Let $P_4 = (x_1, x_2, x_3, x_4)$ and x_5 be the vertex of K_1. There exists a u_i in K_{n-5} is adjacent to x_1 and x_5 (or) If u_i is adjacent to x_1 and u_j for i $\neq$ j is adjacent to x_5. Then in both the cases $\{x_2, x_4, x_5, u_i, u_j\}$ is a γ_2 set. So that n =6, since χ= n–5=1, for which G is totally disconnected. Hence no graph exists. If u_i is a adjacent to x_2 (or equivalently x_3) and x_5. Then $\{x_1, x_3, x_4, x_5, u_i, u_j\}$ is a γ_2 set. So that n = 7. Hence K = K_2 = uv. If u is adjacent to x_2 and x_5, then G ≅ G_1. In all other cases, no new graph exists.

Subcase(c) Let <S>=$K_{1,4}$. Since G is connected. Let the vertex x_1 be adjacent to x_2,x_3,x_4,x_5. There exists a vertex u_i in K_{n-5} which is adjacent to x_1 or any one of $\{x_2,x_3,x_4,x_5\}$. Then $\{x_2,x_3,x_4,x_5,u_i,u_j\}$ for i≠j is a γ_2 set. So that n=7. Hence K=K_2 =uv. If u is adjacent to x_1, then γ_2= 6. Hence G ≅ S ($K_{1,6}$) and if u is adjacent to x_5 then n=6, so that γ_2 = 5, which is a contradiction. Hence no graph exists.

For all the remaining cases, no new graph exists.

Case (iii) Let γ_2 (G) =n-2, χ (G)= n-4, since χ = n-4, G contains a clique K on (n-4) vertices or does not contain a clique K on n-4 vertices. Let S =$\{x_1,x_2,x_3,x_4\}$ ∈ V-S. Then <S>=K_4, $\overline{K}_4$,P_4, $K_3 \cup K_1$, $K_{1,3}$, $K_2 \cup K_2$, $P_3 \cup K_1$

If <S> = K_4, then no graph exists.

Subcase(a) Let <S>= $\overline{K}_4$. Since G is connected, one of the vertices of K_{n-4} says u_i is adjacent to all the vertices of S (or) three vertices of S (or) two vertices of S (or) one vertex of S. Then in all the cases, $\{x_1,x_2,x_3,x_4,u_i,u_j\}$for i≠j is a γ_2 set. So that γ_2=6. Hence n = 8. So that K=K_4. Let u_1,u_2,u_3,u_4 be the vertices of K_4. If all the vertices of S are adjacent to u_1, then γ_2 =6 and $d(x_1) =d(x_2) =d(x_3) =d(x_4) =1$. Hence G ≅ K_4 (4, 0, 0, 0). If three vertices of S are adjacent to u_1 and the fourth one is adjacent to u_2 and $d(x_1) =d(x_2) =d(x_3) =d(x_4) =1$, then γ_2 = 6. Hence G ≅ K_4 (3, 1, 0, 0). If two vertices of S are adjacent to u_1 and the remaining two vertices are adjacent to u_2 and $d(x_1) =d(x_2) =d(x_3) =d(x_4) =1$, then γ_2 =6. Hence G ≅ K_4 (2, 2, 0, 0).

Subcase(b) Let <S>=P_4=(x_1,x_2,x_3,x_4). Since G is connected, there exists a vertex say u_i in K $_{n-4}$ is adjacent to x_1 (or equivalently x_4) (or) x_2 (or equivalently x_3). Let u_i be adjacent to x_1, then $\{x_2,x_4,u_i,u_j\}$ for i≠j is a γ_2 set. So that n= 6. Hence K = K_2 = uv. If x_1 is adjacent to u, then γ_2 =4. Hence G ≅ P_6. Let u_i be adjacent to x_2, then $\{x_1,x_3,x_4,u_i,u_j\}$ for i≠j is a γ_2 set. So that n= 7. Hence K=K_3. Let u_1,u_2,u_3 be the vertices of K_3. If x_1 of S is adjacent to u_1, then γ_2 =4, which is a contradiction. Hence no graph exists. If x_2 is adjacent to u_1 then γ_2 = 5. Hence G ≅ G_2.

Subcase(c) Let <S>= $K_{1,3}$. Let the vertex x_1 be adjacent to x_2,x_3,x_4. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to x_1 or any one of (x_2,x_3,x_4) . Then in both the cases, $\{x_2,x_3,x_4,u_i,u_j\}$ for i≠j is a γ_2 set . So that n = 7.

Hence $K=K_3$. Let u_1,u_2,u_3 be the vertices of K_3. If u_1 is adjacent to x_1 then $\gamma_2 =5$. Hence $G \cong G_3$. If u_1 is adjacent to x_4 then $\gamma_2 = 5$. Hence $G \cong G_4$.

Subcase(d) Let $<S> = P_3 \cup K_1$. Let $P_3 = (x_2,x_3,x_4)$, since G is connected. There exists a vertex say u_i in K_{n-4} which is adjacent to x_1. Again since G is connected, we consider the following two situations: (i) The vertex u_i is adjacent to x_2 (or equivalently x_4) or x_3. (ii)There exists a vertex u_j for $i\neq j$ in K_{n-4} such that u_j is adjacent to x_2(or equivalently x_4)or x_3. Then in all the cases, $\{x_1,x_2,x_4,u_i,u_j\}$ for $i\neq j$ is a γ_2 set. So that n= 7. Hence $K=K_3$. Let u_1,u_2,u_3 be the vertices of K_3. Let u_1 be adjacent to x_1 and x_2 (or equivalently x_3) and let u_1 be adjacent to x_1 and u_2 be adjacent to x_2 (or equivalently x_3). Then in all the cases, $\gamma_2 =5$. Hence $G \cong G_5,G_6,G_7,K_3(1,3,0)$.

Subcase(e) Let $<S> = K_2\cup K_2$. Let x_1x_2 and x_3x_4 be the edges in $<S>$. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to x_1 and x_3 in S (or) u_i is adjacent to x_1 and u_j is adjacent to x_3 for $i\neq j$ in K_{n-4} . Then in both the cases, $\{x_2,x_4,u_i,u_j\}$ for $i\neq j$ is a γ_2 set, hence $\gamma_2 =4$, so that n=6. Hence $K = K_2 = uv$. If u is adjacent to x_1 and x_3 then $\gamma_2 =4$. Hence $G \cong G_8$. If u is adjacent to x_1 and v is adjacent to x_3, then $\gamma_2 =4$. Hence $G \cong P_6$.

Subcase(f) Let $<S> = K_3 \cup K_1$. Since G is connected, there exists a vertex u_i in K_{n-4} is adjacent to x_1 and x_4 (or) u_i is adjacent to x_1 and u_j for $i\neq j$ is adjacent to x_4. Then in both the cases, $\{x_2,x_3,x_4,u_i,u_j\}$ for $i\neq j$ is a γ_2 set of G. So that $\gamma_2 = 5$. Hence n= 7. Since χ = n-4 =3. Hence $K=K_3$. Let u_1,u_2,u_3 be the vertices of K_3. If u_1 is adjacent to x_1 and x_4, then $\gamma_2 = 5$. Hence $G \cong G_9$. If u_1 is adjacent to x_1 and u_2 is adjacent to x_4 then $\gamma_2 =4$, which is a contradiction. Hence no graph exists.

Case (iv) Let γ_2 = n-3 & χ = n-3, Since G is connected. Since χ = n-3, G contains a clique K on (n-3) vertices or does not contain a clique K on n-3 vertices. Let $S =\{x_1,x_2,x_3\} \in V\text{-}S$. Then $<S> =K_3, \overline{K}_3, P_3, K_2 \cup K_1, P_2 \cup K_1$.

Subcase(a) Let $<S> = K_3$. Since G is connected, let x_1 be adjacent to u_i for some i in K_{n-3}. Then $\{x_2,x_3,u_i,u_j\}$ for $i\neq j$ is a γ_2 set . So that γ_2= 4implies that n=7. Hence $K = K_4$. Let u_1,u_2,u_3,u_4 be the vertices of K_4 . If u_1 is adjacent to x_1 then $\gamma_2 =4$. Hence $G \cong G_{10}$.

Subcase(b) Let $<S>= \overline{K}_3$. Since G is connected, one of the vertices of K_{n-3} says u_i is adjacent to all the vertices of S (or) two vertices of S (or) one vertex of S. Then in all the cases, $\{x_1,x_2,x_3,u_i,u_j\}$for $i\neq j$ is a γ_2 set. So that n=8. Hence $K=K_5$. Let u_1,u_2,u_3,u_4,u_5 be the vertices of K_5, without loss of generality, u_1 is adjacent to all the vertices of S and $d(x_1)=d(x_2)=d(x_3)=1$, then $\gamma_2 =5$. Hence $G \cong K_5(3, 0, 0, 0, 0)$. If u_1 is adjacent to x_1, x_2 and u_2 is adjacent to x_3 and $d(x_1) = d(x_2) = d(x_3) = 1$, then $\gamma_2 =5$. Hence $G \cong K_5(2, 1, 0, 0, 0)$. If u_1 is adjacent to x_1 and u_2 is adjacent to x_2 and u_3 is adjacent to x_3, then $\gamma_2 = 5$. Hence $G \cong K_5(1, 1, 1, 0, 0)$.

Subcase(c) Let $<S> = P_3 =(x_1,x_2,x_3)$. Since G is connected. There exists an vertex u_i in K_{n-3} is adjacent to x_1 (or equivalently x_3) (or) u_i is adjacent to x_2 . Then in both the cases, $\{x_1,x_3,u_i,u_j\}$ for $i\neq j$ is a γ_2 set. So that n = 7. Hence $K = K_4$. Let u_1,u_2,u_3,u_4 be the vertices of K_4. If u_1 is adjacent to x_1 then $\gamma_2 =4$. Hence $G \cong K_4(P_4)$. If u_1 is adjacent to x_2 then $\gamma_2 = 4$. Hence $G \cong G_{11}$.

Subcase(d) Let $<S>= K_2 \cup K_1$. Let x_1x_2 be the edge in K_2. Since G is connected.

There exists an u_i in K_{n-3} is adjacent to x_1 and x_3 (or) u_i is adjacent to x_1 and u_j for $i \neq j$ is adjacent to x_3. Then in both the cases, $\{x_2,x_3,u_i,u_j\}$ for $i \neq j$ is a γ_2 set. So that $\gamma_2 = 4$, implies that n =7. Hence $K=K_4$. Let u_1,u_2,u_3,u_4 be the vertices of K_4. If u_1 is adjacent to x_1 and x_3, then $\gamma_2 = 4$ and $d(x_1) = 2$ and $d(x_2) = d(x_3) = 1$. Hence $G \cong G_{12}$. If u_1 is adjacent to x_1 and u_2 is adjacent to x_3, then $\gamma_2 = 4$. Hence $G \cong G_{13}$.

Case (v): Let $\gamma_2 = n-4$ and $\chi = n-2$, since $\chi = n-2$, G contains a clique K on (n-2) vertices or does not contain a clique K on n-2 vertices. Let $S = \{x_1,x_2\} \in V-S$. Then <S> $=K_2$ or $\overline{K}_2$.

Subcase(a) Let<S>$=K_2$. Since G is connected, there exists a vertex u_i in K_{n-2} is adjacent to x_1. Then $\{x_2,u_i,u_j\}$ for $i \neq j$ is a γ_2 set . So that n=7. Hence $K = K_5$. Let u_1,u_2,u_3,u_4,u_5 be the vertices of K_5. If u_1 is adjacent to x_1, then $\gamma_2 = 3$ and $d(x_1) = 2$, $d(x_2) = 1$. Hence $G \cong K_5(P_3)$.

Subcase(b) Let <S>= $\overline{K}_2$. Since G is connected, there exists a vertex u_i in K_{n-2} is adjacent to x_1 and x_2 (or) If u_i is adjacent to x_1 and u_j for $i \neq j$ is adjacent to x_2. Then in both the cases, $\{x_1,x_2,u_i,u_j\}$ for $i \neq j$ is a γ_2 set. So that n=8. Hence $K = K_6$. Let u_1,u_2,u_3,u_4,u_5,u_6 be the vertices of K_6. If x_1 and x_2 be adjacent to u_1, then $\gamma_2 = 4$ and $d(x_1) = d(x_2) = 1$. Hence $G \cong K_6(2, 0, 0, 0, 0, 0)$. If x_1 is adjacent to u_1 and x_2 is adjacent to u_2, then $\gamma_2 = 4$ and $d(x_1) = d(x_2) = 1$. Hence $G \cong K_6(1,1,0,0,0,0)$.

Case (vi) Let $\gamma_2 = n-5$ and $\chi = n-1$, since $\chi = n-1$, G contains a clique K on (n-1) vertices or does not contain a clique K on n-1 vertices. There exists a vertex u_i in K_{n-1} is adjacent to x. Then $\{x,u_i,u_j\}$ for $i \neq j$ is a γ_2 set . So that n= 8. Hence $K = K_7$. Let $u_1,u_2,u_3,u_4,u_5,u_6,u_7$ be the vertices of K_7. If u_1 is adjacent to x then $\gamma_2 = 3$. Hence $G \cong K_7(1,0,0,0,0,0,0)$.

Case (vii) Let $\gamma_2 = n-6$ and $\chi = n$, since $\chi = n$ then $G = K_n$, G must be complete. But for K_n, $\gamma_2 = 2$. So that n=8. Hence $G \cong K_8$.

References

[1] Harary F (1972): Graph Theory, Addison Wesley Reading Mass.

[2] Haynes, Teresa W. (2000): Paired domination in graphs, congr. Numer 150

[3] Haynes, Teresa W. (2001): Induced-Paired domination in graphs, Arts combin. 57, 111-128.

[4] Teresa W .Haynes, Stephen T. Hedemiemi and Peter J .Slater (1998): Fundamentals of domination in graphs, Marcel Dekker, Newyork.

[5] Mahadevan.G and Mydeenbibi.A (2007): Double domination number and Chromatic number of a graph,Mathematical model and computational models, Narosa publications,, India.P.182-190.

[6] Mahadevan G. (2005): On Domination theory and related concepts in graphs, Ph.D., thesis Manonmaniam Sundaranar University, Tirunelveli, India.

[7] Paulraj Joseph J .and Mahadevan G.(2003): Complementary connected domination number and chromatic number of a graph, Referred Proceedings of the second National Conference on Mathematical and computational Models, edited by Arulmozhi and Natarajan, Allied publications, India. 56P. 342-349.
[8] Paulraj Joseph J. and Mahadevan.G. (2006): On Complementary perfect domination number of a graph, Acta Cienia Indica, Vol. XXXI M, No, 2, 847
[9] Mahadevan G, Selvam A, Paulraj Joseph J (2005): Extended results on Complementary connected domination number and chromatic number of a graph ,proceedings of the national conference SACOEFERENCE conducted by Dr.Sivanthi Aditanar college of Engineering, Tiruchendur, India.P.438-442.
[10] Paulraj Joseph J. and Arumugam S (1992): Domination and connectivity in graphs, International Journal of Management and systems, Vol 8, No 3:233-236.
[11] Paulraj Joseph J. and Arumugam S (1999): Domination and Colouring in graphs, International Journal of management and systems, Vol 15, No.1, 37-44
[12] Paulraj Joseph J. and Mahadevan G (2006). Paired Domination and Chromatic number of a graph, International Journal of Management and systems.