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Satisfactory Marriage Problem

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Abstract

The present paper describes an algorithm, which will determine Satisfactory matching of classical Satisfactory Marriage problem based on preference value, using Hungarian algorithm of assignment model.

Introduction

The stable marriage problem(SM) was introduced by Gale and Shapley (1962) [1].An SM instance consists of two finite equal sized set of members, called men and women. Each man m_i ($1 \leq i \leq n$, n is the number of men) prefer women in strict order, forming his own preference list. Similarly, each woman w_j ($1 \leq j \leq n$) prefer men in strict order, forming her own preference list [2].A stable marriage is a one to one matching of the men with the women such that there is no man-woman pair that prefers each other over their present mates. Gale and Shapley describe a simple algorithm (G.S. algorithm) [3] which, form stable matching based on the given arbitrary preference list.

Gale and Shapley showed that at least one stable matching exists for every SM [4]. However, in general there are many different stable matching for a single instance, and the G.S. algorithm finds only one of them (man-optimal or woman-optimal) with an extreme property. In the man-optimal stable matching, each man is matched with his best possible partner, while each woman gets her worst possible (or if we exchange the role of men and women, the resulting Matching is woman-optimal) [5]. Hence, it is natural to try to obtain a matching which is not only stable but also “good” in some criterion.

The paper is organized as follows. In the next section, classical Satisfactory Marriage Problem (STMP), then preliminary definitions to describe an algorithm, then an algorithm to solve STMP and finally, result is discussed with an example.

Classical Satisfactory Marriage Problem

The Classical Satisfactory Marriage problem is a problem of satisfactory matching. An STMP instance consists of two finite equal sized set of members, called men and women. Each man m_i prefer women in strict order, forming his own preference list. Similarly, each woman w_j prefer men in strict order, forming her own preference list. A satisfactory marriage is a one to one matching of the men with the women such that difference between satisfactory value of man and woman in man-woman pair is comparatively low.

For the related terminologies such as Preference value, Men's and Women's preference value, Men's and Women's preference value matrix, Satisfactory value matrix, Satisfactory level refer [7] and Optimal matching (Satisfactory matching) refer [8].

Example: 2.1

Consider an STMP instance with three men m_1 , m_2 , m_3 and three women w_1 , w_2 , w_3.The preference lists of men and women are given below in the order of preference.

m_1: w_2 w_3 w_1 w_1: m_1 m_2 m_3

m_2: w_1 w_2 w_3 w_2: m_1 m_3 m_2

m_3: w_2 w_1 w_3 w_3: m_2 m_1 m_3

The preference value of w_2, w_3, w_1 with respect to m_1 is 3/3,2/3 and 1/3, preference value of w_1, w_2, w_3 with respect to m_2 is 3/3,2/3 and 1/3 and preference value of w_2, w_1, w_3 with respect to m_3 is 3/3,2/3 and 1/3 .

Now the men's preference value matrix is

$$PM_M = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{1}{3} & \frac{3}{3} & \frac{2}{3} \\ \frac{3}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{3}{3} & \frac{1}{3} \end{pmatrix} \end{array}$$

Similarly the women's preference value matrix is

$$PM_W = \begin{array}{c} \\ w_1 \\ w_2 \\ w_3 \end{array} \begin{array}{c} \begin{array}{ccc} m_1 & m_2 & m_3 \end{array} \\ \begin{pmatrix} \frac{3}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{3}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{3}{3} & \frac{1}{3} \end{pmatrix} \end{array}$$

The satisfactory value matrix is,

$$SM_M = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{4}{3} & \frac{6}{3} & \frac{4}{3} \\ \frac{5}{3} & \frac{3}{3} & \frac{4}{3} \\ \frac{3}{3} & \frac{5}{3} & \frac{2}{3} \end{pmatrix} \end{array} \qquad SM_W = \begin{array}{c} \\ w_1 \\ w_2 \\ w_3 \end{array} \begin{array}{c} \begin{array}{ccc} m_1 & m_2 & m_3 \end{array} \\ \begin{pmatrix} \frac{4}{3} & \frac{5}{3} & \frac{3}{3} \\ \frac{6}{3} & \frac{3}{3} & \frac{5}{3} \\ \frac{4}{3} & \frac{4}{3} & \frac{2}{3} \end{pmatrix} \end{array}$$

It is observed that $SM_M = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{4}{3} & \frac{6}{3} & \frac{4}{3} \\ \frac{5}{3} & \frac{3}{3} & \frac{4}{3} \\ \frac{3}{3} & \frac{5}{3} & \frac{2}{3} \end{pmatrix} \end{array} = SM_W^T$

About Assignment Model

The assignment Model [6] is one of the fundamental combinatorial optimization models in the branch of operations research in mathematics. In its most general form, if there are a number of agents and a number of tasks that are to be assigned to agents. Any agent can be assigned to perform any task, incurring some cost that may vary depending on the agent-task assignment. The objective of this model is to assign all tasks on one to one basis to all agents in such a way that the total cost of the assignment should be minimized/maximized. To resolve this type of assignment problem Hungarian method is used.

In matching problem the objective is to match (assign) every man in the men group with every woman in the women group on one to one basis. In order to get the maximum possible satisfaction for each member in both groups (sex), find the optimal assignment of men to women on one to one basis to maximize the satisfaction level.

Hence assignment method will give the optimal matching with maximum satisfactory level. Hereunder an algorithm to find satisfactory matching for any STMP instance is described using assignment method. For Hungarian algorithm refer [8].

Satisfactory Matching Algorithm (SMA)

1. Get number of men (m) or women (w), say n
2. Get the preference lists of all men and women
3. Assign preference value for each w_j with respect to each m_i, according to men's preference list and construct PM_M
4. Assign preference value for each m_i with respect to each w_j, according to women's preference list and construct PM_W
5. Form a satisfactory value matrix $SM_{M/W}$,adding PM_M and transpose of PM_W (or PM_W and transpose of PM_M)
6. Apply Hungarian method to find optimal matching for both men and women such that the total assignment value should be maximized. That indicates optimal satisfactory level of both.

Example 4.1: Consider the SM_M of example 2.1 to find best matching with maximum satisfactory level using the given algorithm.

$$SM_M = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{4}{3} & 2 & \frac{4}{3} \\ \frac{5}{3} & 1 & \frac{4}{3} \\ 1 & \frac{5}{3} & \frac{2}{3} \end{pmatrix} \end{array}$$

The Satisfactory matching on applying SMA algorithm, is (m_1,w_3), (m_2,w_1) and (m_3,w_2) and satisfactory value for each pair is given in the following table.

Satisfactory matching	Satisfactory value	Satisfactory level of men	Satisfactory level of women
(m_1,w_3)	4/3	m_1=2/3	w_3=2/3
(m_2,w_1)	5/3	m_2=1.0	w_1=2/3
(m_3,w_2)	5/3	m_3=1.0	w_2=2/3

The above result shows that the satisfactory level of any member in both groups is minimum 2/3.The matching obtained is the best matching for both groups and the satisfactory level of each group is optimum.

Conclusion

Gale and Shapley showed that at least one stable matching exists for every SM instance with an extreme property. In the man-optimal stable matching, each man is matched with his best possible partner, while each woman gets her worst possible, among all stable matching. In this paper, we gave an algorithm for STMP instance to find Satisfactory matching, which is optimal for both men and women as for as satisfactory level of group is concern, using assignment technique. That is n members in both group gets a maximum possible satisfactory level with respect to the given Preference list and that will be the best possible matching.

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