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An improvement in LQR controller design for TLIP system via model order reduction

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Abstract

The Model order reduction is crucial for analysing complex order systems. The primary objective of reduced order modelling aims at obtaining the lower order equivalent which gives sufficient representation of its higher order counterpart while preserving the key features of the system. The controller designed for such reduced order systems is simpler in terms of its dimensionality and controller parameters involved as compared to the same for higher order system. This paper focusses on using Particle Swarm Optimization (PSO) reduced model for comparing the end results of LQR controller which has been designed for both higher and lower order equivalents of Triple Link Inverted Pendulum (TLIP) system. The reduced order model uses a controller which is simplified and gives better performance due to decreased effect of certain control parameters on the overall controller design.

Keywords – controller, inverted pendulum, linear quadratic regulator, Model order reduction, Particle Swarm Optimization.

I. INTRODUCTION

The analysis of practical system becomes complex when the order of the system increases. Simplification of such systems while preservation of the key features makes the analysis computationally easier without much loss in its efficiency [1]. This is Model Order Reduction (MOR) which attempts to capture the most essential features of a structure without disturbing the desired outcome. Therefore, there exists its need for due to a number of reasons. Primarily, the complexity in modelling which hinders its usage in real problems. Further, higher order systems leads to large computations that are cumbersome and time taking for evaluating various factors that are to be controlled during the analysis.

The design of the controllers has been greatly impacted by model reduction techniques, leading to reduced numerically complicated procedures in solving practical problems. The performance of the system has been improved by simplification of the dynamic models of the systems containing many equations and/or variables, leading in reduction of time taken during simulations and storage requirements and still providing reliable outcome. This would provide the designer with simpler controllers which are much reduced dimensionally, that may have less hardware requirements and thus leading to much less intricate analysis.

MOR can be achieved by numerous methods that are available in literature [2-7], but Routh Approximation and Pade technique were most commonly used. Other such methods include Dominant pole, Balanced truncation, Stability equation method, pole clustering etc. Recently, Evolutionary Algorithms have seen a tremendous increase in its application for model order reduction due to their direct application and software simulation techniques available. They use processes inspired by natural biological evolution, like reproduction, mutation, recombination and natural selection [8]. The increasing popularity of Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) techniques gave way to the discovery of many other nature inspired techniques like, Ant Colony Optimization, Artificial Bee Colony, and Firefly Algorithms. PSO inspired by swarm intelligence, gained its popularity due to its simple evaluation procedure. The system is initialized by a population of particles in a swarm which represents candidate solutions and then, iteratively, optimum value of fitness function is evaluated in search space by updating generations, to find the optimized value of objective function or reduced order model in MOR [9-13].

The basic benchmark control system of Inverted Pendulum, is the theoretical implementation of real life systems such as still standing position of a human being where the central nervous system gives stability, or a robotic arm and even walking robots where the crux lies in stabilising the various links together. The three basic structures; single arm pendulum, cart inverted pendulum and double inverted pendulum [14] can be modelled to be used as experimentation platform for controller design. The three links in TLIP system results in four eighth order transfer functions which can be reduced to lower order by various techniques [9-10].

This paper demonstrates the ease in controller design for reduced order model as compared to that of its higher order model, by considering the Triple Link Inverted Pendulum system as the test model. The reduced order model for this system has been taken from [9-10]. LQR controller is designed for both its higher and lower order equivalents and the results are compared to validate the improvement in controller design and analysis.

II. LQR CONTROLLER DESIGN

Any controller such as PID, Neural or Fuzzy can be designed for achieving certain performance in TLIP system [16-21]. In this paper we demonstrate LQR controller design for both original and simplified models to achieve certain same performance measure in both the systems. LQR controller is preferred for its simplicity and direct use in MATLAB environment. The implementation of LQR controller is given in figure 1. Before designing the LQR controller for TLIP model, the certain characteristics of model like stability, controllability and observability, are to be analysed for adequate system performance.

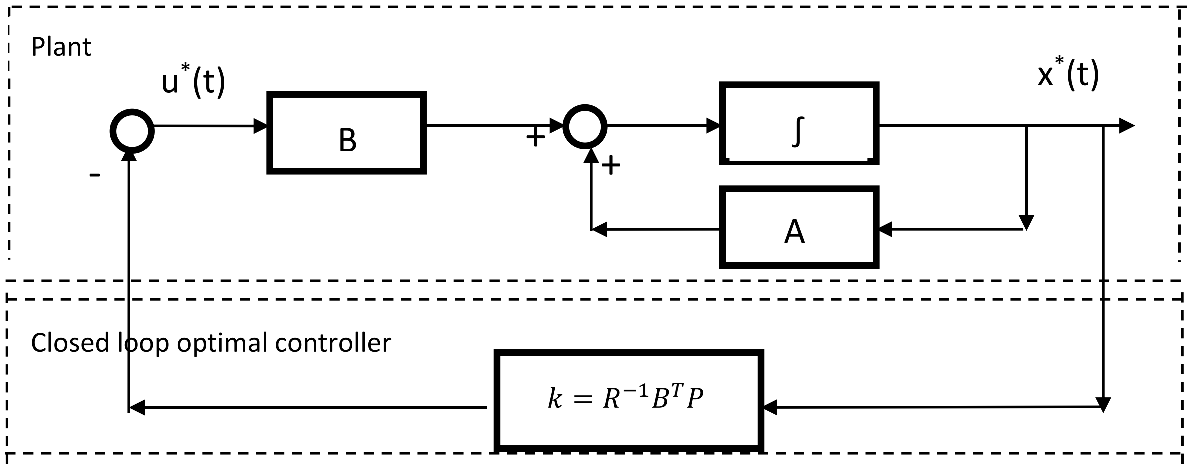

Figure 1. Implementation of LQR controller [18].

LQR is an optimal controller, based on state feedback method. With all controllable states, it minimizes the performance index of a linear system given as [22-24];

$$\dot{x} = Ax + Bu \tag{1}$$

For the linearized system given in equation 1, with infinite final time, the quadratic Performance Index (PI) is given as:

$$J = \frac{1}{2}\int_0^\infty [x^T(t)Q(t)x(t) + u^T(t)R(t)u(t)]dt \tag{2}$$

The PI is to be minimized by the control input,u given as,

$$u = -kx \tag{3}$$

where,

$$k = R^{-1}B^TP \tag{4}$$

and Q and R are positive semi definite matrices which are to be user defined.

Q and R, given in equation 2, are error weight matrices which constitute the relative effect of states and control inputs respectively to the final objective function of the given system. For efficient LQR controller design, these matrices are to be selected to effectively minimize the performance index (PI) given in as *J*. For the TLIP system, the system outputs, which are; x, θ_1, θ_2, θ_3 [16] are the main variables to be controlled and analysed.

The matrices Q and R are chosen to be diagonal so that the objective function *J* is kept squared positive, given as;

$$J = q_1 x_1^2 + q_2 x_2^2 + \cdots + r_1 u^2 \tag{5}$$

The LQR technique used in this work gives effective system performance and is used in designing a controller for the higher and lower order TLIP system to achieve required performance specifications.

III. SIMULATION RESULTS

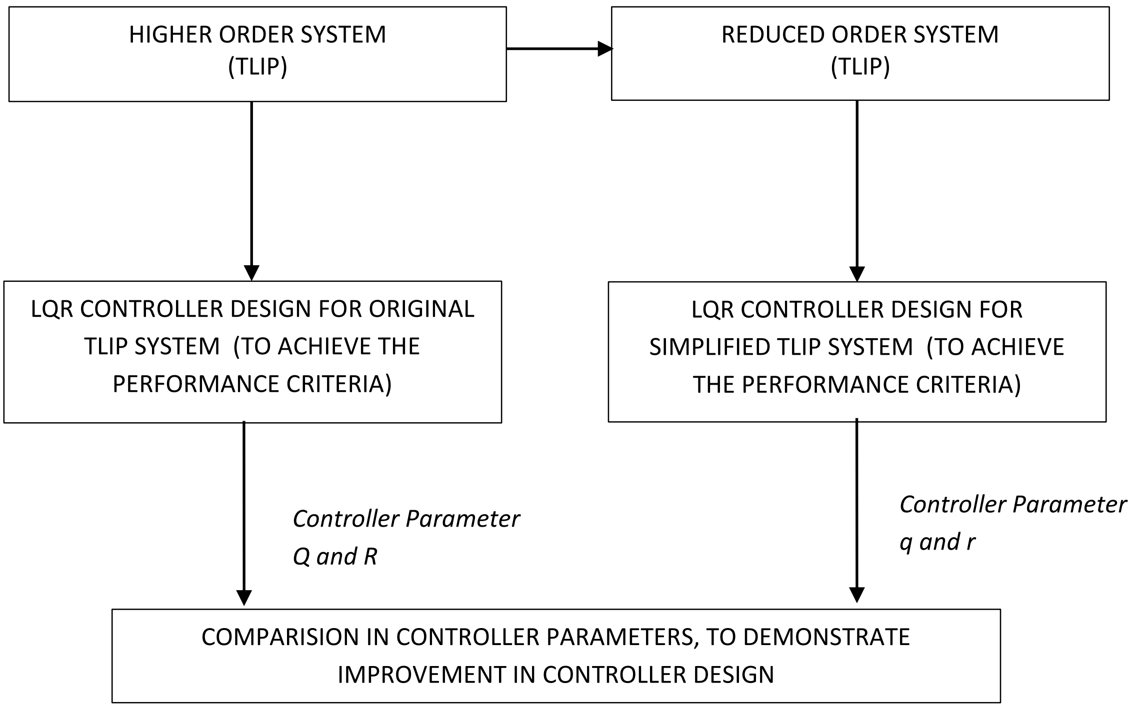

Figure 2: Flow of process. It shows the procedure followed to demonstrate the improvement in controller design.

The higher order TLIP model is a single-input, multiple-output system which offers four different transfer functions whose stabilized transfer functions in MATLAB environment are obtained from [9]. These complex 8^{th} order transfer functions are reduced to third order equivalents by using PSO technique by minimising the error between the higher and lower order counterparts in time domain as taken from [9].

To demonstrate the improvement in controller design using model order reduction, LQR controller is designed for both TLIP higher and reduced order models so as to achieve a set of performance specifications as listed in Table 1. The flow of process to demonstrate improved controller design is given in figure 2.

Table 1: Controller Design Specifications

S No.	Controlled Variable	Settling Time(sec)	Peak Amplitude
1.	x	2.52	0.0140
2.	θ_1	2.74	0.0070
3.	θ_2	2.40	0.0042
4.	θ_3	2.50	0.0028

Controller for higher order TLIP model

Q and R matrices are to be selected in diagonal form as;

$$Q=\text{diag}\ ([Q_{11}\ Q_{22}\ Q_{33}\ Q_{44}\ 0\ 0\ 0\ 0]) \tag{6}$$

$$R=r_{11} \tag{7}$$

These are selected iteratively by the user to achieve the desired criterion. For the TLIP system, several different weighting matrices were tried and tested to achieve the required performance as in table 2. The elements of the Q and R matrices of the LQR selected for the system under consideration are; Q_{11}=500, Q_2 =5000, Q_{33}= 5000, Q_{44} = 5000, R = 1. Note that angular control weights are dominant to displacement control weight.

The matrix R gives weight to the input voltage whereas, Q weights displacement of cart and the angular position of the links. The elements of the Q matrix are selected to be larger than that of the R matrix. This selection demonstrates relative importance of states as compared to input so as to achieve stability. The optimal controller design problem is computed in MATLAB environment by evaluation of state feedback control parameters K and P using LQR function;

[K, P, E] = lqr (A, B, Q, R)

where, E is the open loop Eigen value.

The optimal feedback gain matrix is;
K = [3.3703 -75.8589 99.1906 134.6078 24.0046 8.2370 27.3314 14.7853].
The resulting closed loop system is given as;

$$\dot{X} = (A - BK)X \quad (8)$$

The controller output response to achieve the performance specifications as given in Table 1 is shown in figure 3 for the four higher order transfer functions of TLIP system.

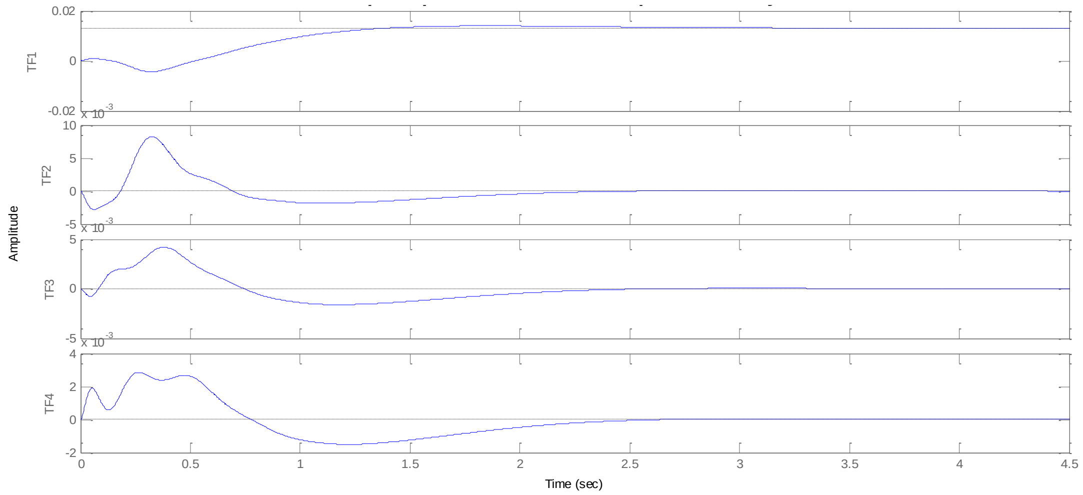

Figure 3. Controller Responses for original TLIP System.

Controller for reduced order TLIP model

The Q and R matrices selected for the four transfer functions of TLIP to achieve the required performance measures based on hit and try are given in table 2 as follows;

$$Q=\text{diag}([q_{11}\ q_{22}\ q_{33}]) \quad (9)$$

$$R=r \quad (10)$$

Table 2: LQR controller's Q and R matrices for reduced order TLIP model

Transfer function	q_{11}	q_{22}	q_{33}	r
x/u	450	620	40	14
Θ_1/u	20500	0	200	100
Θ_2/u	13500	500	0	10
Θ_3/u	4000	0	49000	10

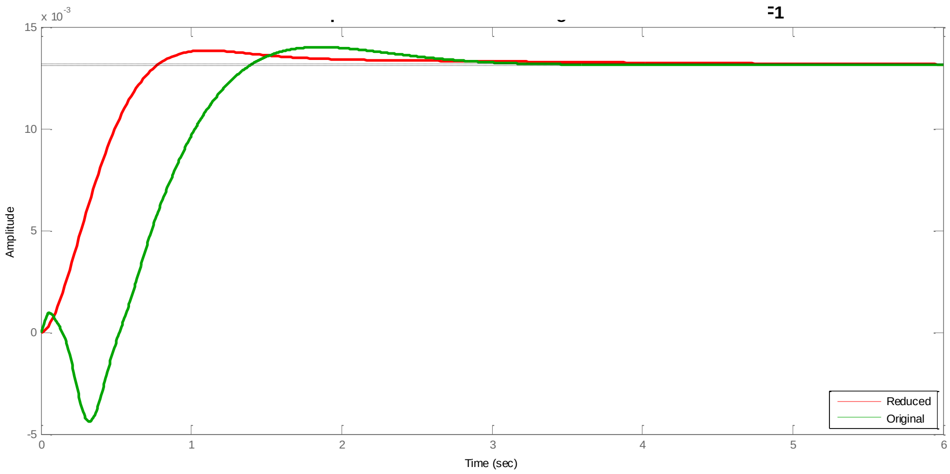

Figure 4: Controller Responses for reduced and original transfer function TF1

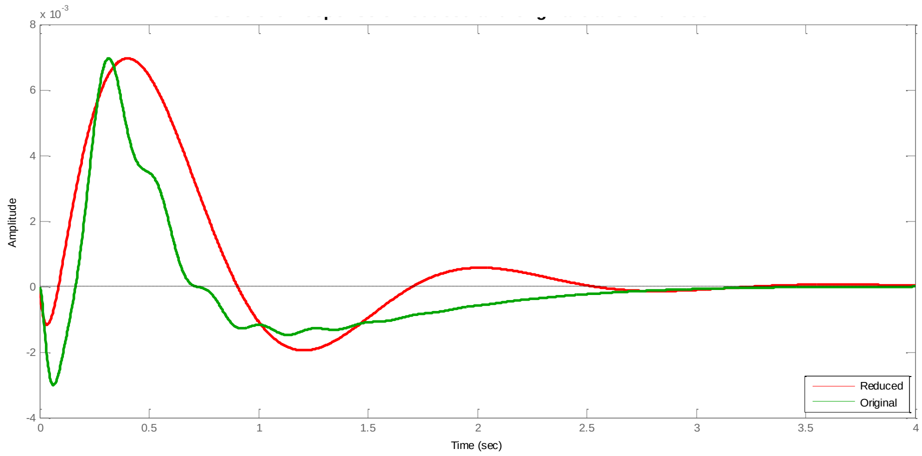

Figure 5: Controller Responses for reduced and original transfer function TF2

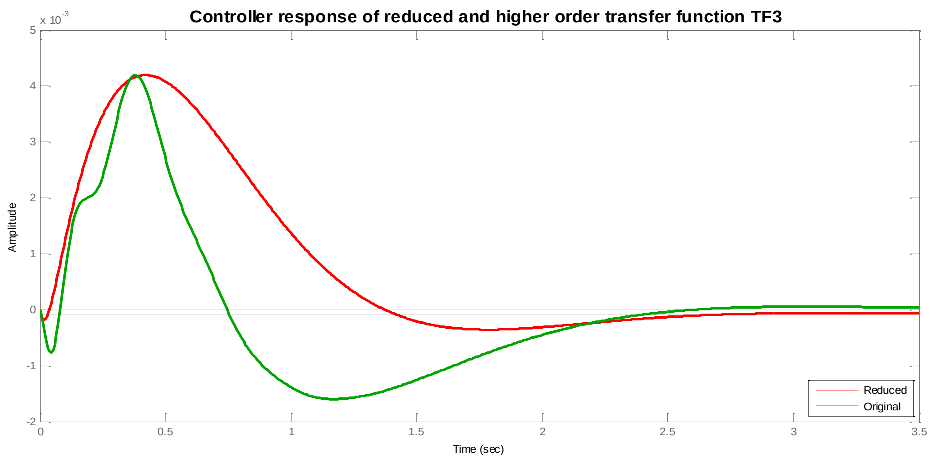

Figure 6: Controller Responses for reduced and original transfer function TF3

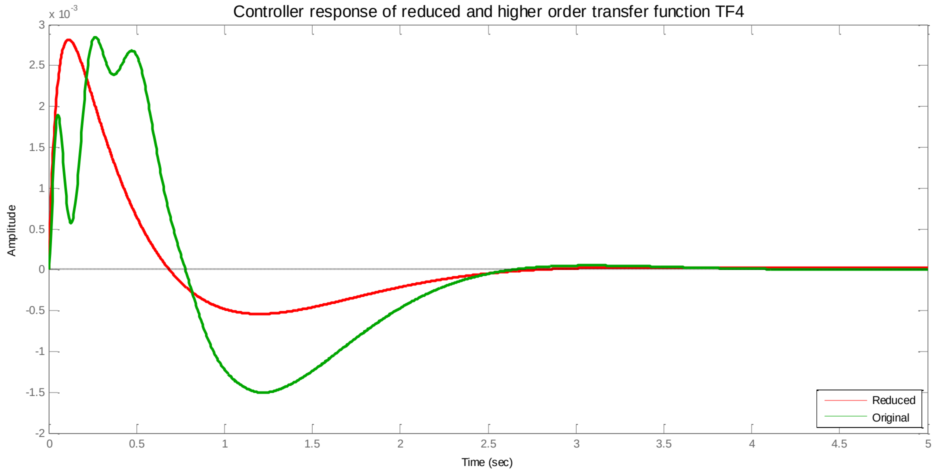

Figure 7: Controller Responses for reduced and original transfer function TF4

The output response of the controller designed for reduced order transfer functions to achieve the performance specifications as given in Table 1 is plotted in figures 4-7 where the controller responses for original as well as reduced order transfer functions is shown. Both the controllers have been designed to achieve same specifications so that the comparison can be made for analysing the simplicity in design of controller for reduced order system.

The controllers designed for simplified models are of lesser order as those for original higher order systems. This gives reduced hardware requirements. It can also be seen from the LQR controller designed for higher order model and reduced order model of TLIP in figures 4-7 that with same performance indices achieved from both models, table 2 shows that some of the terms in Q matrices are missing. It clearly demonstrates that their corresponding control variables do not influence the overall objective function J (as given in equation 5). Due to the reduced or negligible contribution of some variables in the objective function, the controller designed for lower order system is less susceptible to errors from certain variables hence, more effective controller design and more robust system.

IV. CONCLUSION

By designing a controller for higher as well as lower order models of TLIP system to achieve the same performance, the improvement in controller design for lower order model can easily be seen from the controller parameter matrices. An improvement in controller design by model order reduction can be seen by comparing the controllers designed for both higher and lower order equivalents of TLIP system. MOR not only gives a reduced order controller for simplified TLIP system but also the influence of some states of the system can further be ignored without much loss in properties. The complexity of controller is greatly reduced and lesser no. of states are to be controlled to minimize J. Thus, reduced order modelling reduces the complexity of controller design.

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