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A Matrix Pencil Method For the Efficient Computation of Direction of Arrival Estimation for Weakly Correlated Signals Using Uniform Linear Array in a Low SNR Regime

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ABSTRACT

A Matrix Pencil method using a single snapshot is used to compute the Direction of arrival by measuring the incident angles down from the perpendicular direction along an antenna array. The Multiple Signal Classification (MUSIC) algorithm is used as a benchmark. The correlation coefficient of the signals is computed by varying the phase of the source signals at which they impinge on the Uniform Linear Array and from which a measure is obtained to distinguish between weakly and highly correlated signals. Results from computer simulations show that the Root Mean Square Error (RMSE) of Matrix Pencil (MP) and MUSIC method decreases with increasing Signal to Noise Ratio (SNR) and angle separation of source signals. The MP method provides more advantages over MUSIC algorithm at low Signal to Noise Ratio (SNR).The variance of the Matrix Pencil method estimate approaches the Cramér–Rao bound (CRB) indicating that MP method outperforms MUSIC algorithm during low SNR. In the investigations through simulations it was established that the Matrix Pencil method exhibited robust stability as another characteristic across all correlation coefficients in terms of Root Mean Square Error. This consequently shows that a lower computational cost can be achieved without being affected by the correlation coefficient of the source signals at low SNR for a better sensitivity in the Direction of Arrival finder system.

Keywords - Angle of elevation, Correlation Coefficient, Direction of Arrival, Multiple Signal Classification (MUSIC), and Matrix Pencil.

I. INTRODUCTION

The problem of Direction of Arrival estimation has been considered as an important problem in signal processing in various fields such as radio astronomy, sonar, radar and wireless communication. Many techniques were suggested to solve the problem such as Capon's minimum variance technique which attempts to solve the poor resolution problems which are associated with the delay-and-sum method [1], [2]. More advanced techniques have been proposed that are based on the eigen-structure of the input covariance matrix including multiple signal classification (MUSIC), Root-MUSIC and Estimation of Signal Parameters via Rotational Invariance Techniques(ESPRIT).These provide high resolution DOA estimation[3]. The Music algorithm proposed by Schmidt [4] returns the pseudo-spectrum at all frequency samples. Root-MUSIC [5] returns the estimated discrete frequency spectrum, along with the corresponding signal power estimates.

Furthermore, in some environments such as those that possess rapid change in fading characteristics, assumptions on ergodicity may lose validity because the correlation matrix estimation may be poor. These techniques may involve modification of the covariance matrix for highly correlated signals using a preprocessing scheme called spatial smoothening although this increases the computational complexity [6]. Thus, Direction of Arrival estimation in the presence of multipath is a challenging task. These limitations have led to new techniques such as Matrix Pencil [4] which uses low computational load as the estimation of correlation matrix is not required. Increasing the accuracy as well as reducing the computational complexity is also a vital requirement in real time systems.

Matrix Pencil method is based on the spatial samples of the data which is done on a snapshot basis [7]. Both the conventional covariance matrix techniques and Matrix Pencil method can estimate the Direction of Arrival easily on Uniform Linear Array when arranged in an order to avoid aliasing [8]. However, these two algorithms have a problem of progressive degradation of their performance with reduced Signal to Noise Ratio. This has led to more research to be done in a comparative analysis under Low SNR with low computation complexity. However, the degree of stability needs to be determined across various correlation coefficients of both algorithms.

II. MATHEMATICAL MODEL FOR ULA

The uniform linear array (ULA) is one of the most convenient mathematical model which consists of antenna elements which are aligned and equally spaced along a line. Its simplicity and regularity makes it advantageous for array processing. The manifold vector of ULA allows its application in several powerful techniques used in array processing

If there exists P isotropic sources emitting narrowband signals in the far field of the array such that P plane signals with additive white Gaussian noise are impinging on the array. Source signals impinge on ULA of M elements with separation of d .The signal model will be deduced from [9] to be:

$$X(t) = As(t) + n(t) \tag{1}$$

Where $\boldsymbol{X}(t)$ is the observed signal vector, $\boldsymbol{S}(t)$ is the source signal vector, $n(t) = \left[n_1(t)n_2(t)...n_M(t)\right]^T$ is the noise vector, $\boldsymbol{A} = \left[a(\theta_1),...,a(\theta_P)\right]$ is the steering vector and $s(t) = \left[s_1(t)s_2(t)...s_P(t)\right]$.Also the signals received signal at all antenna elements can also be expressed as in equation (2) where $a(\theta)$ is the steering vector and c is the speed of light .

$$X(t) = \begin{bmatrix} x_1(n) \\ x_2(n) \\ . \\ . \\ . \\ x_M \end{bmatrix} = \begin{bmatrix} 1 \\ e^{-j\omega\frac{d\sin\theta_1}{c}} \\ \\ \\ \\ e^{-j\omega\frac{(M-1)d\sin\theta_1}{c}} \end{bmatrix} s(t) \tag{2}$$

$$= a(\theta)s(t)$$

The narrowband signal $S_p(t)$ is assumed to be the received signal wavefront with $s_p(t)$ and ω_p as its complex envelope and angular frequency respectively as seen in equation.

$$S_p(t) = s_p(t)\exp\left\{j\omega_p(t)\right\} \tag{3}$$

III. MATRIX PENCIL ALGORITHM

The Matrix Pencil (MP) method is a direct data domain method that can be used to estimate direction of arrivals (DOA) of correlated and uncorrelated signals. Although most popular methods are based on the estimation of the signal correlation which require several snapshots of the incoming signals. The Matrix Pencil method can estimate the accurate DOA in a single snapshot. In addition, the signals are assumed to be uncorrelated which may not be the case in a practical situation especially in wireless communications.

A Matrix Pencil method in [10] consists of $\left\{\hat{Y}_a, \hat{Y}_b\right\}$ matrices were λ is a solution which satisfy the matrix of the form:

$$\det\left(\hat{Y}_b - \lambda \hat{Y}_a\right) = 0 \tag{4}$$

When noise is taken into consideration, a form of an approximation of the Hankel matrix $\hat{Y}$ from the components, sampled at time for $i = 1,...,M$, of the observed signal vector as $x_i\left(t\right)$ from [6].

$$\hat{Y} = \begin{bmatrix} x_1(t) & x_2(t) & . & . & . & x_L(t) \\ x_2(t) & x_3(t) & . & . & . & x_{L+1}(t) \\ . & & . & & & . \\ . & & & . & & . \\ . & & & & . & . \\ x_{M-L+1}(t) & x_{M-L+2}(t) & . & . & . & x_M(t) \end{bmatrix}_{(M-L+1)\times(L)} \tag{5}$$

The parameter L is also known as the *Pencil parameter*. For the interest of efficient noise filtering, it is best to choose L between the values $M/3$ and $M/2$ [6] where M the number antenna elements in the Uniform Linear Array is. Using singular value decomposition, the noisy Hankel matrix $\hat{Y}$ can be written as:

$$\hat{Y} = U \sum V^H \tag{6}$$

Where $U = \left[u_1 \quad . \quad . \quad . \quad u_{M-L+1}\right]$ and $V = \left[v_1 \quad . \quad . \quad . \quad v_L\right]$ as in [11] are unitary matrices with columns that are the eigenvectors of YY^H and $Y^H Y$ respectively. The $\sum$ corresponds to matrix with the singular values $\sigma_1 \geq \sigma_2,...\sigma_{\min}$ which are descending along the diagonal of $\hat{Y}$ described in [12]. These can be used to estimate P by setting it to the largest i for which $\sigma_i / \sigma_{\max} = 10^{-r}$, where r is the number of significant decimal digits. From [6], the decomposition of $\hat{Y}$ can be broken down in to two subspaces $\hat{Y} = U_S \sum_S V_S^H + U_N \sum_N V_N^H$ which correspond to the signal and noise subspace. The singular values and vectors corresponding to the noise subspace are discarded leaving a truncated matrix.

$$\hat{Y}_T = U_S \sum_S V_S^H \tag{7}$$

To form a pencil, matrices $\hat{Y}_a$ and $\hat{Y}_b$ are created in a manner by deleting the last and first rows of $\hat{Y}_T$ respectively. This helps to create the pencil $\left\{\hat{Y}_b - \lambda \hat{Y}_a\right\}$. As stated in [9], the solutions of the pencil λ can thus be found as the generalized eigenvalues of the matrix pair $\left\{\hat{Y}_a, \hat{Y}_b\right\}$ or equivalently as the eigenvalues of $\left\{\hat{Y}_a^{\#}\hat{Y}_b - \lambda I\right\}$ where $\hat{Y}_a^{\#}$ is the pseudoinverse of $\hat{Y}_a$ defined by $\hat{Y}_a^{\#} = \left(\hat{Y}_a^H \hat{Y}_a\right)^{-1} \hat{Y}_a^H$. For noisy data when λ is set to z_i , $\lambda = z_1, z_2, ..., z_P$ and solve for $z_i = \lambda_i$.

Directions of arrival are then extracted from the eigenvalues $z_i = \lambda_i$ as:

$$\theta_i = \sin^{-1}\left(\frac{im\{\log_e(z_i)\}}{\frac{2\pi}{\eta_0} d}\right) \tag{8}$$

Where η_0 is the wavelength and im represents the imaginary.

SUMMARY OF MATRIX PENCIL

Step i

The radiation pattern from the sources is first sampled to form a discrete pattern data set. Then the discrete data set are organized in a form of $\hat{Y}$ which is a Hankel matrix.

Step ii

The Hankel matrix $\hat{Y}$ undergoes a Singular Value Decomposition (SVD). Singular values and vectors corresponding to the noise subspace are discarded to leave a truncated matrix $\hat{Y}_T = U_S \sum_S V_S^H$.

Step iii

Two sub-matrices are generated from truncated matrix defined as $\hat{\boldsymbol{Y}}_a$ and $\hat{\boldsymbol{Y}}_b$ .These matrices are formed by deleting last and first row respectively on the truncated matrix.

Step iv

The pencil is created to form $\{\hat{Y}_b - \lambda \hat{Y}_a\}$ which is used to solve for a set of eigen values $\lambda = z_1, z_2, ..., z_P$.

Step v

Directions of arrival are then extracted from the eigenvalues using equation *(8)*

IV. MUSIC ALGORITHM

The MUSIC algorithm uses the spatial covariance matrix which undergoes Eigen-decomposition to separate the signal and noise subspaces. This becomes easier if the signals are weakly correlated since the signal and noise subspace are orthogonal to each other [13]. The estimated DOA in [14] is produced based on the orthogonality of the noise subspace to the signal subspace and thus formation of pseudo frequency spectrum.

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta) E_N E_N^H(\theta)} \tag{9}$$

The denominator of equation is the product of the signal vector $a(\theta)$ and the noise matrix E_N which are orthogonal to each other. The above equation gives results for the Direction of Arrival of the source signal by searching through the peak values on the spectrum. The highest P peaks corresponds the angles under estimation. For highly correlated signals, MUSIC algorithm fails to estimate the DOA and therefore it can be improved by introducing an identity transitional matrix I as in [15] such that the received vector matrix becomes J from X as seen in equation *(1)*.Thus: $J = X^* I$ *(10)*

where X^* is the complex conjugate of the X which is the received signal This is also known as spatial smoothening .

V. SIMULATIONS AND RESULTS

In this section, the simulation results are given to illustrate the performance of different algorithms. MATLAB is used for simulating the algorithms in terms of RMSE and variance. Therefore in order to measure the similarities between two incident source signals from equation *(3)*, the phase defines the correlation coefficient using;

$$\rho = \frac{E\{S_1(t)\} E\{S_P(t)\}}{\sqrt{E\{|S_1(t)^2|\} E\{|S_P(t)^2|\}}} \tag{11}$$

Where $|\rho| \le 1$ and $E\{\bullet\}$ is the expectation operator which is the inner product representing the vectors that contain $S_1(t)$ and $S_2(t)$ samples respectively.

The $|S_1(t)^2|$ and $|S_2(t)^2|$ represent the square magnitude corresponding to the vector Euclidean norm such that in this case the correlation coefficient geometrically represents the phase angle between the vectors of $S_1(t)$ and $S_2(t)$. This concludes that,

$$-1 \le \rho = \frac{S_1(t) \bullet S_p(t)}{\sqrt{|S_1(t)|^2 |S_p(t)|^2}} = \frac{S_1(t) \bullet S_p(t)}{|S_1(t)||S_p(t)|} \triangleq \cos(S_1(t), S_P(t)) \le 1 \quad .$$

The correlation coefficient is obtained from the source signals by varying the phase between the two source signals as they impinge along the antenna array. This is calculated through distinguished angular frequencies of the two signals. The signals are considered highly correlated when the correlation coefficient is equal or close to minus one. Even though they are represented in opposite direction (negative) but they have same sample values. For weakly correlated signals, correlation coefficient will be close to zero.

From the range, a correlation coefficient of approximately $\rho = 1$ is assumed to be highly correlated incident signals and $\rho = 0.3$ for weakly correlated signals. The Matrix Pencil Algorithm is compared against MUSIC algorithm for ULA with and without spatial smoothening. This is done in both weakly and highly correlated environment in all the simulations.

In addition, the estimator performance was evaluated using the Root Mean Square Error (RMSE).For two incident signals with θ_1 .and θ_2 are evaluated as:

$$RMSE = \frac{\sqrt{E\left\{\left(\theta_1 - \hat{\theta}_1\right)^2\right\}} + \sqrt{E\left\{\left(\theta_2 - \hat{\theta}_2\right)^2\right\}}}{2} \tag{12}$$

Where θ = actual angle and $\hat{\theta}$ = estimated angle.

Another metric measurement is the variance of the two estimators in equation *(13)*. This was used to obtain the closeness of the estimator to the as derived in [17].The Mean Square Error (MSE) or variance is obtained as:

$$Variance = \frac{E\left\{\left(\theta_1 - \hat{\theta}_1\right)^2\right\} + E\left\{\left(\theta_2 - \hat{\theta}_2\right)^2\right\}}{2} \tag{13}$$

The simulations which were conducted are all under the assumption that the number of incident signals are predetermined with some common parameters which are shown in Table 1.

Table 1. Common parameters for the simulations

Parameter	Symbol	Value
Number of antenna elements	*M*	*8*
Pencil parameter	*L*	4
DOA,Source 1	θ_1	20^0
DOA,Source 2	θ_2	60^0
Number of trials		*1000*

A. Weakly Correlated Signals

In figure 1 and figure 2, improved MUSIC and MUSIC are compared with matrix Pencil algorithm respectively.

Two incident signals are considered with correlation coefficient of $\rho = 0.3$ are considered Matrix Pencil algorithm outperform MUSIC algorithm. Although the conventional MUSIC is known for good performance in weakly correlated environment, it is affected by limited number of snapshots which is not the case Matrix Pencil [7].

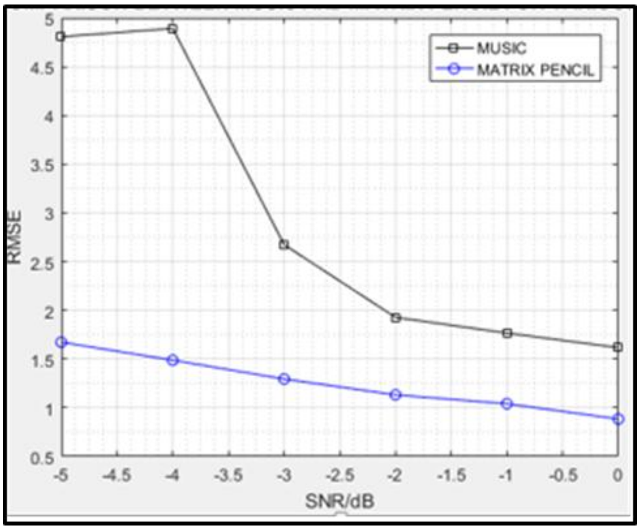

Figure 1. RMSE vs SNR for Improved MUSIC and Matrix Pencil algorithm

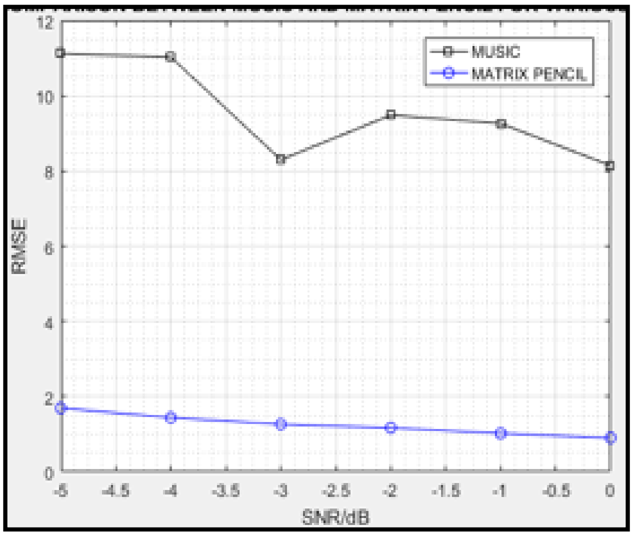

Figure 2. RMSE vs SNR, for MUSIC and Matrix Pencil algorithm.

The section below for figure 3 and figure 4 shows results for RMSE vs. angular separation of the two sources. The position of Source 1 remained fixed at 20^0 while the position of Source 2 was varied from 21^0 up 35^0. The simulations were obtained at low SNR of $-5dB$. Both algorithms tend to fail although Matrix Pencil is seen to outperform MUSIC algorithm in both figures. Although spatial smoothening is applied to MUSIC algorithm, the Matrix pencil method still give superior results.

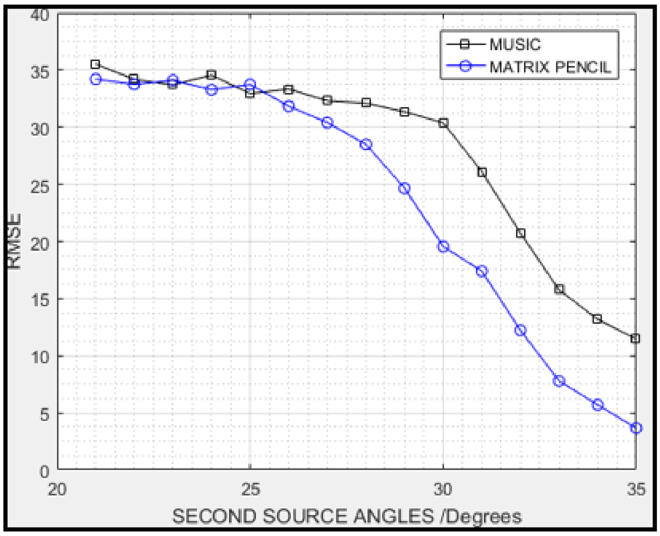

Figure 3. RMSE vs SNR, for Angle separation for Improved MUSIC and Matrix Pencil algorithm

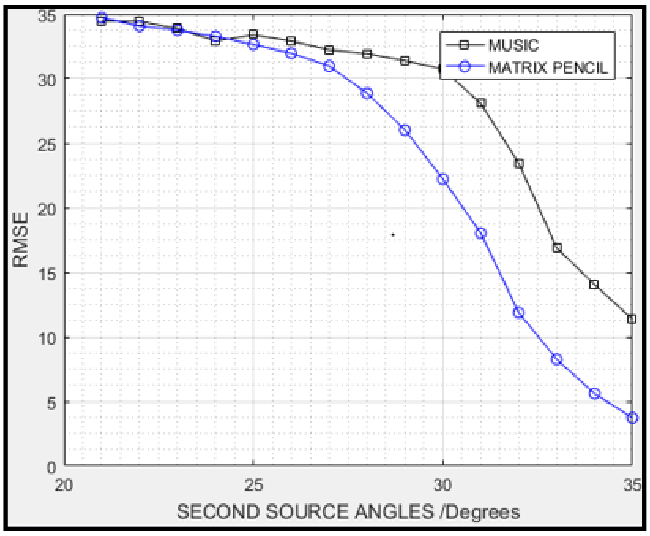

Figure 4. RMSE vs SNR for Angle separation for MUSIC and Matrix Pencil algorithm

B. Highly Correlated Signals

While varying the SNR as seen in figure 5 and figure 6, there are two source signals still used with correlation coefficient of $\rho = 1$. The Matrix Pencil method still shows better results than MUSIC algorithm. Using the conventional MUSIC algorithm for highly correlated signals increases the RMSE than it was before in the weakly correlated environment. The Matrix algorithm still outperforms the improved MUSIC algorithm.

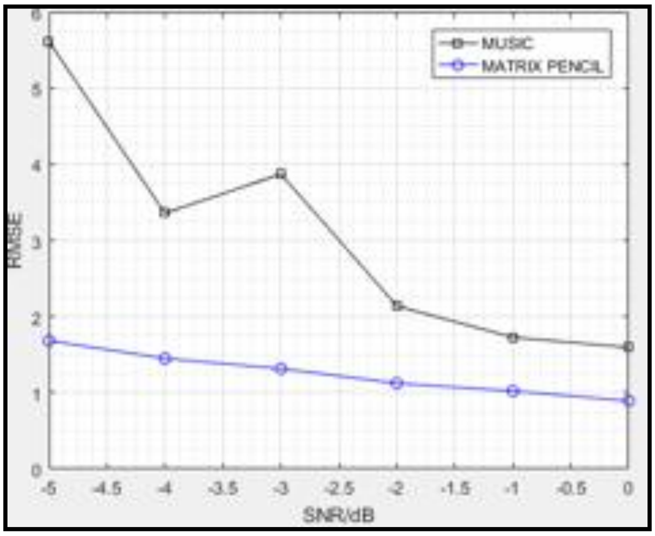

Figure 5. RMSE vs SNR for Improved MUSIC and Matrix Pencil algorithm

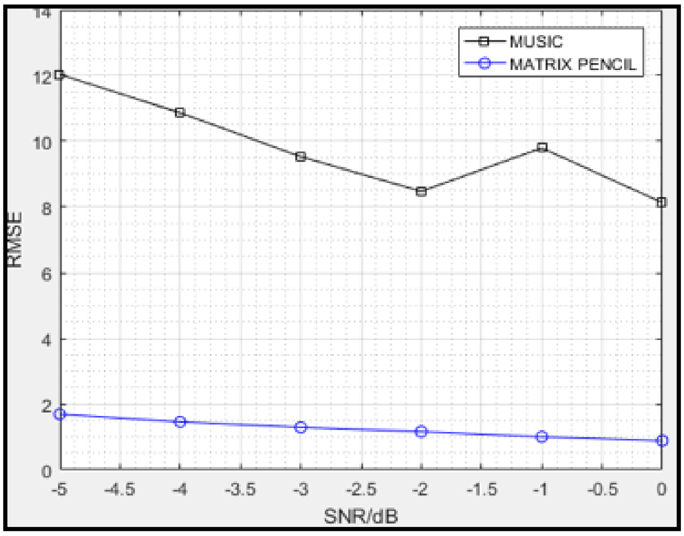

Figure 6. RMSE vs SNR, for MUSIC and Matrix Pencil algorithm

The simulation results in figure 7 and figure 8 are carried out at $-5dB$ of Signal to Noise Ratio with correlation between the signals being 1.The position of Source 1 remained fixed at 20^0 while the position of Source 2 was variable from 21^0 up to 35^0 .The results deduce that the signal subspace fails to be filtered out from noise subspace for both algorithms at very low SNR. The Matrix Pencil still gives better results as separation increases estimator performance vs. angular separation of the two sources.

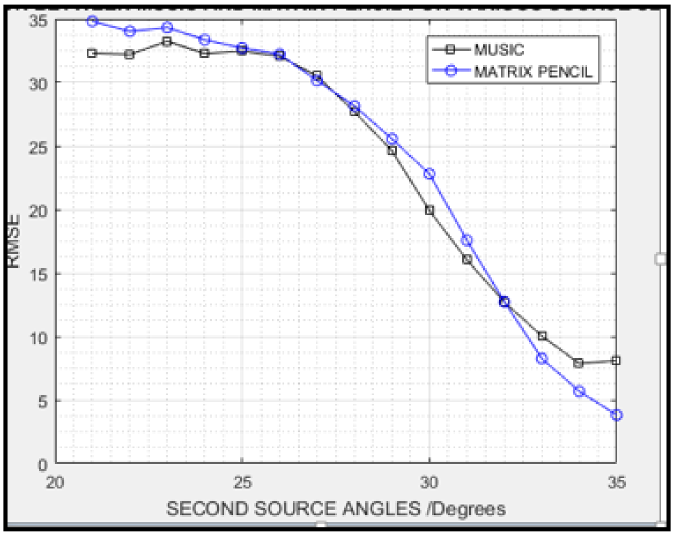

Figure 7. RMSE vs Angle separation for Improved MUSIC and Matrix Pencil algorithm

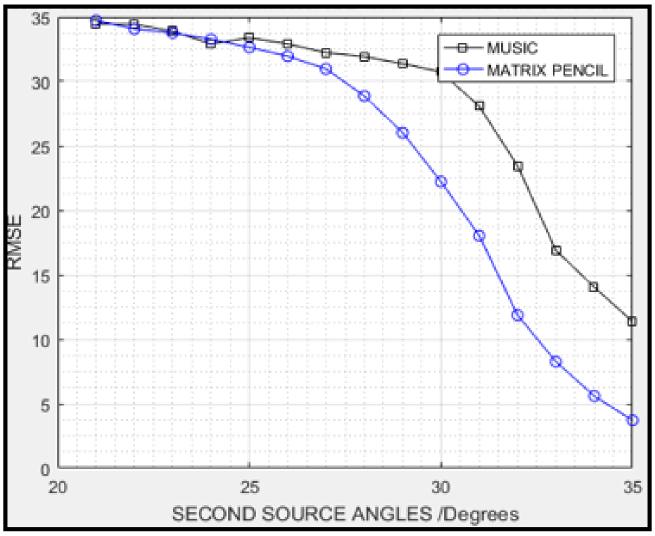

Figure 8. RMSE vs, Angle separation for MUSIC and Matrix Pencil algorithm

VI. PERFORMANCE ON CRAMÉR–RAO BOUND AND CORRELATION COEFFICIENT

Sensitivity property is one of the main requirements in direction estimation systems due its ability to give accurate results. The best sensitivity which is possible to be attained primarily may depend on factors such as measurement time which may be due to the computation complexity of the DOA estimator, noise factor among others [18]. Therefore this all together can be considered as Cramér–Rao bound which are of great interest in practice when estimating the limit. For this reason, the Cramer-Rao Bound can be expressed as the minimum limit in relation to the variance of the estimates made containing Additive White Gaussian Noise.

This section provides a comparison of the variance of the Matrix Pencil method and the improved MUSIC algorithm at very low SNR using highly correlated signals. This was conducted against the Cramér–Rao bound. The results are shown in figure 9. Matrix Pencil algorithm almost achieves the Cramér–Rao bound stochastic for estimator variance especially as Signal to Noise Ratio increases. The Cramer-Rao Bound derived in [17] mainly depends on the N and M representing the number of snapshots and the number of elements on the antenna array respectively.

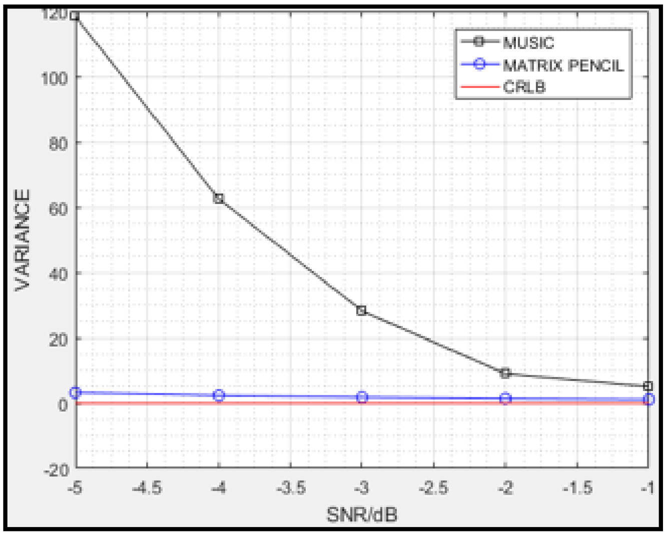

Figure 9. Variance vs SNR for Improved MUSIC and Matrix Pencil algorithm

In figure 10, simulations were conducted under an environment with varying correlation coefficient with Signal to Noise Ratio of $-5dB$. The results of Matrix Pencil algorithm were exhibiting a property of stability with RMSE algorithm across all correlation coefficients.

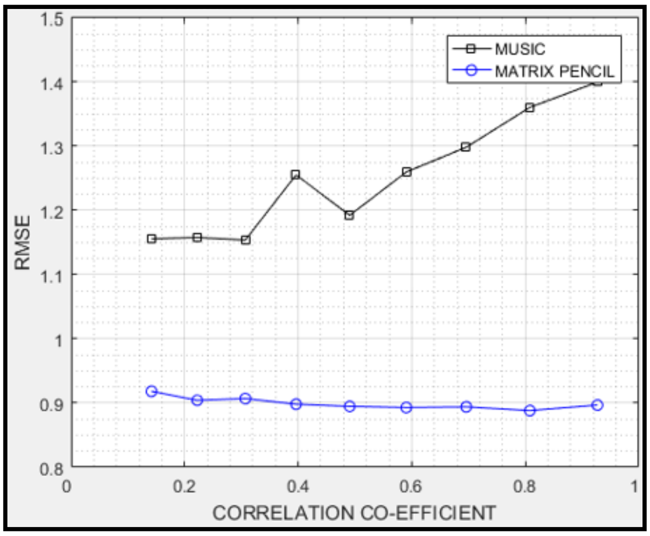

Figure 10. RMSE vs correlation coefficient for MUSIC and Matrix Pencil algorithm

VII. CONCLUSION

The Matrix Pencil algorithm uses eigenvalues obtained from the determinant of the two pencil matrices formed from the signal subspace after Singular Value Decomposition to estimate the Direction of Arrival of incoming signals. The signal subspace can be efficiently filtered out from the noise when the signals imping on the array are both correlated and non-correlated. The RMSE behaviour of Matrix Pencil is relatively constant for both weakly and highly correlated environment exhibiting the degree of accuracy and stability in all environments unlike the MUSIC algorithm. The MUSIC algorithm shows a larger variance than Matrix Pencil algorithm especially at lower Signal to Noise Ratio before it becomes asymptotic to Cramér–Rao bound the variance. The Matrix Pencil algorithm is important in real time system due to its property of robust stability exhibited with a low computational complexity demonstrated across different correlation coefficients. Therefore this makes the Matrix Pencil method to contribute to a good sensitivity in a direction estimation system.

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