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On Object-oriented Concepts in a Soft Context Defined by a Soft Set

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ABSTRACT

We have combined the formal contexts with the soft sets to form so-called soft contexts and introduced the notion of soft concepts. The purpose of this work is to introduce a new type of soft concept (called m-concept or object oriented soft concept) based on soft sets, which is independent of the notion of soft concepts in a soft context but they are closely related to each other and the object oriented concept in formal context. In particular, we study the basic properties of the m-concept and the structure of the set of all m-concepts. Finally, we study how to find all the m-concepts in a soft context.

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1. INTRODUCTION

FCA (formal concept analysis) was introduced by Wille in 1982 [9], which is an important theory for the research of information structures induced by a binary relation between the set of attributes and objects attributes. The three basic notions of FCA are formal context, formal concept, and concept lattice. A formal context is a kind of information system, which is a tabular form of an object-attribute value relationship [2, 3, 8]. A formal concept is a pair of a set of objects as called the extent and a set of attributes as called the intent.

The concept of soft set was introduced by Molodtsov in 1999 [7], to deal complicated problems and uncertainties. The operations for the soft set theory was introduced by Maji et al. in [4]. In [1], Ali et al. proposed new operations modified some concepts introduced by Maji. We have formed a soft context by combining the concepts of the formal context and the soft set defined by the set-valued mapping in [6]. And we introduced and studied the new concepts named soft concepts and soft concepts lattices.

In [10], Yao introduced a new concept called *an object oriented formal concept* in a formal context by using the notion of approximation operations.

We recall that: Let (U, A, I) be a formal context in formal concept analysis, where U is a finite nonempty set of objects, A is a finite nonempty set of attributes and I is a binary relation between U and A. For $x \in U$ and $y \in A$, if $(x, y) \in I$, also written as xIy. We will denote $xI = \{y \in A | xIy\}$; and $Iy = \{x \in U | xIy\}$.

And, let us consider two set-theoretic operations,

$^\Box : P(U) \to P(A)$: $X^\Box = \{y \in A | \forall x \in U(xIy \Rightarrow x \in X)\}$;

$^\Diamond : P(A) \to P(U)$: $Y^\Diamond = \{x \in U | \exists y \in A(xIy \wedge y \in Y)\}$.

Then a pair (X, Y), $X \subseteq U, Y \subseteq A$, is called *an object oriented formal concept* if $X = Y^\Diamond$ and $Y = X^\Box$.

Based on the above facts, we are trying to study a new type derived from a soft concept based on soft-sets. So, in this paper, we are going to introduce and investigate the new notions of object-oriented soft concepts (simply, m-concepts) which is related closely each other and the object oriented concept in formal context. Firstly, we study the notion of m-concepts and basic properties.

2. PRELIMINARIES

A formal context is a triplet (U, A, I), where U is a non-empty finite set of objects, A is a nonempty finite set of attributes, and I is a relation between U and A. Let (U, A, I) be a formal context. For a pair of elements $x \in U$ and $y \in A$, if $(x, y) \in I$, then it means that object x has attribute y and we write xIy. The set of all attributes with a given object $x \in U$ and the set of all objects with a a given attribute $y \in A$ are denoted as the following [8,9]:

$$x^* = \{y \in A | xIy\};\ y^* = \{x \in U | xIy\}.$$

And, the operations for the subsets $X \subseteq U$ and $Y \subseteq A$ are defined as:

$$X^* = \{y \in A | \textit{for all } x \in X, xIy\}; \quad Y^* = \{x \in U | \textit{for all } y \in Y, xIy\}.$$

In a formal context (U, A, I), a pair (X, Y) of two sets $X \subseteq U$ and $Y \subseteq A$ is called a *formal concept* of (U, A, I) if $X = Y^*$ and $B = Y^*$, where X and Y are called the *extent* and the *intent* of the formal concept, respectively.

Let U be a universe set and A be a collection of properties of objects in U. We will call A *the set of parameters* with respect to U.

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A pair (F, A) is called a *soft set* [7] over U if F is a set-valued mapping of A into the set $P(U)$ of all subsets of the set U, i.e.,

$$F : A \to P(U).$$

In other words, for $a \in A$, every set $F(a)$ may be considered as the set of a-elements of the soft set (F, A).

Let $U = \{z_1, z_2, \dots, z_m\}$ be a non-empty finite set of *objects*, $A = \{a_1, a_2, \dots, a_n\}$ a non-empty finite set of *attributes*, and $F : A \to P(U)$ a soft set. Then the triple (U, A, F) is called *a soft context* [6].

And, in a soft context (U, A, F), we introduced the following mappings: For each $Z \in P(U)$ and $Y \in P(A)$,
(1) $\mathbf{F}^+ : P(A) \to P(U)$ *is a mapping defined as* $\mathbf{F}^+(Y) = \cap_{y \in Y} F(y)$;
(2) $\mathbf{F}^- : P(U) \to P(A)$ is a mapping defined as $\mathbf{F}^-(Z) = \{a \in A : Z \subseteq F(a)\}$;
(3) $\Psi : P(U) \to P(U)$ is an operation defined as $\Psi(Z) = \mathbf{F}^+\mathbf{F}^-(Z)$.

Then Z is called a *soft concept* [6] in (U, A, F) if $\Psi(Z) = \mathbf{F}^+\mathbf{F}^-(Z) = Z$. The set of all soft concepts is denoted by $sC(U, A, F)$.

3. MAIN RESULTS

Definition 3.1. Let (U, A, F) be a soft context. Then for $C \in P(A), X \in P(U)$,

an operator $\mathbb{F} : P(A) \to P(U)$ is defined by $\mathbb{F}(C) = \cup_{c \in C} F(c)$;

an operator $\overleftarrow{\mathbb{F}} : P(U) \to P(A)$ is defined by $\overleftarrow{\mathbb{F}}(X) = \{c \in A : F(c) \subseteq X\}$.

Simply, we denote: For $c \in A$ and $x \in U$ $\mathbb{F}(\{c\}) = \mathbb{F}(c)$ and $\overleftarrow{\mathbb{F}}(\{x\}) = \overleftarrow{\mathbb{F}}(x)$. Obviously, $\mathbb{F}(c) = F(c)$ for $c \in A$.

Theorem 3.2. *Let (U, A, F) be a soft context, $S, T \subseteq U$ and $B, C \subseteq A$. Then we have:*

(1) If $S \subseteq T$, then $\overleftarrow{\mathbb{F}}(S) \subseteq \overleftarrow{\mathbb{F}}(T)$; if $B \subseteq C$, then $\mathbb{F}(B) \subseteq \mathbb{F}(C)$;

(2) $\mathbb{F}\overleftarrow{\mathbb{F}}(S) \subseteq S$; $\overleftarrow{\mathbb{F}}\mathbb{F}(B) \subseteq B$;

(3) $\overleftarrow{\mathbb{F}}(S \cap T) = \overleftarrow{\mathbb{F}}(S) \cap \overleftarrow{\mathbb{F}}(T)$, $\mathbb{F}(B \cup C) = \mathbb{F}(B) \cup \mathbb{F}(C)$;

(4) $\overleftarrow{\mathbb{F}}(S) = \overleftarrow{\mathbb{F}}\mathbb{F}\overleftarrow{\mathbb{F}}(S)$, $\mathbb{F}(B) = \mathbb{F}\overleftarrow{\mathbb{F}}\mathbb{F}(B)$.

Proof. Obvious. □

Example 3.3. Let $U = \{1, 2, 3, 4\}$ and $A = \{a, b, c, d, e\}$. Consider a soft context (U, A, F) as Table 1.

Then we can get the soft set (F, A) induced by a set-valued mapping $F : A \to P(U)$ as follows:

$F(a) = F(b) = \{1, 2, 3\};\ F(c) = \{1, 2, 4\};\ F(d) = \{1, 3\};\ F(e) = \{1\}.$

So, the following things are obtained:

Table 1: A soft context

-	a	b	c	d	e
1	1	1	1	1	1
2	1	1	1	0	0
3	1	1	0	1	0
4	0	0	1	0	0

(1) For $X = \{1, 3, 4\}$, $\mathbb{F}\overleftarrow{\mathbb{F}}(X) = \mathbb{F}(\{d, e\}) = \{1, 3\}$. So, $\mathbb{F}\overleftarrow{\mathbb{F}}(X) \neq X$.

(2) For $C = \{a, b\}$, $\overleftarrow{\mathbb{F}}\mathbb{F}(C) = \overleftarrow{\mathbb{F}}(\{1, 2, 3\}) = \{a, b, d, e\}$. So, $\overleftarrow{\mathbb{F}}\mathbb{F}(C) \neq C$.

(3) For $X = \{1, 2, 4\}$ and $Y = \{1, 3, 4\}$, $\overleftarrow{\mathbb{F}}(X) \cup \overleftarrow{\mathbb{F}}(Y) = \{c, d, e\}$ and $\overleftarrow{\mathbb{F}}(X \cup Y) = U$. So, $\overleftarrow{\mathbb{F}}(X) \cup \overleftarrow{\mathbb{F}}(Y) \neq \overleftarrow{\mathbb{F}}(X \cup Y)$.

(4) For $C = \{d, e\}$ and $D = \{b, e\}$, $\mathbb{F}(C) \cap \mathbb{F}(D) = \{1, 3\}$, $\mathbb{F}(C \cap D) = \{1\}$. So, $\mathbb{F}(C) \cap \mathbb{F}(D) \neq \mathbb{F}(C \cap D)$.

Definition 3.4. Let (U, A, F) be a soft context. For each $X \in P(U)$,

$\mathfrak{F} : P(U) \to P(U)$ is an operator defined by $\mathfrak{F}(X) = \mathbb{F}\overleftarrow{\mathbb{F}}(X)$,

where

$C = \overleftarrow{\mathbb{F}}(X) = \{c \in A : F(c) \subseteq X\}; \mathbb{F}(C) = \cup_{c \in C} F(c)$.

Theorem 3.5. *Let (U, A, F) be a soft context. Then we have:*

(1) $\mathfrak{F}(X) \subseteq X$ for $X \subseteq U$.

(2) If $X \subseteq Y$, then $\mathfrak{F}(X) \subseteq \mathfrak{F}(Y)$.

(3) $\mathfrak{F}(\mathfrak{F}(X)) = \mathfrak{F}(X)$ for $X \subseteq U$.

Proof. It is obvious from Theorem 3.2. □

Remark 3.6. Let (F, X) be a soft set over a universe set U. As shown in the next example, for $X, Y \in P(U)$,

$$\mathfrak{F}(X \cap Y) \neq \mathfrak{F}(X) \cap \mathfrak{F}(Y); \quad \mathfrak{F}(X) \cup \mathfrak{F}(Y) \neq \mathfrak{F}(X \cup Y).$$

Example 3.7. Let $U = \{1, 2, 3, 4, 5\}$ and $A = \{a, b, c, d, e, f\}$. Consider a soft context (U, A, F) where a set-valued mapping $F : A \to P(U)$ is defined by

$$F(a) = F(d) = \{1, 2, 4\};\ F(b) = \{2, 4, 5\};$$

$$F(c) = \{2, 4\};\ F(e) = F(f) = \{1, 3, 5\}.$$

(1) For $X = \{1, 2, 4\}$ and $Y = \{5\}$, $\mathfrak{F}(X \cup Y) = \mathfrak{F}(\{1, 2, 4, 5\}) = \{1, 2, 4, 5\}$, $\mathfrak{F}(X) \cup \mathfrak{F}(Y) = \{1, 2, 4\}$. So, $\mathfrak{F}(X \cup Y) \neq \mathfrak{F}(X) \cup \mathfrak{F}(Y)$.

(2) For $X = \{1, 2, 4\}$ and $Y = \{1, 3, 5\}$, $\mathfrak{F}(X \cap Y) = \mathfrak{F}(\{1\}) = \emptyset$, $\mathfrak{F}(X) \cap \mathfrak{F}(Y) = \{1, 2, 4\} \cap \{1, 3, 5\} = \{1\}$. So, $\mathfrak{F}(X \cap Y) \neq \mathfrak{F}(X) \cap \mathfrak{F}(Y)$.

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In [10], Yao introduced a new concept called *an object oriented formal concept* in a formal context by using the notion of approximation operations.

We recall that: Let (U, A, I) be a formal context in formal concept analysis, where U is a finite nonempty set of objects, A is a finite nonempty set of attributes and I is a binary relation between U and A. For $x \in U$ and $y \in A$, if $(x, y) \in I$, also written as xIy, we say that x has the property y, or the property y is possessed by object x.

For an object $x \in U$, the set of properties of x is denoted by:

$$xI = \{y \in A | xIy\}.$$

For a property $y \in A$, the set of objects of y is denoted by:

$$Iy = \{x \in U | xIy\}.$$

For the formal context (U, A, I), let us consider two set-theoretic operators, $^\Box : P(U) \to P(A)$ and $^\Diamond : P(A) \to P(U)$:

$$\begin{aligned} X^\Box &= \{y \in A | \forall x \in U(xIy \Rightarrow x \in X)\} \\ &= \{y \in A | Iy \subseteq X\}; \end{aligned}$$

$$\begin{aligned} Y^\Diamond &= \{x \in U | \exists y \in A(xIy \wedge y \in Y)\} \\ &= \{x \in U | xI \cap Y \neq \emptyset\} \\ &= \cup_{y \in Y} Iy. \end{aligned}$$

A pair (X, Y), $X \subseteq U, Y \subseteq A$, is called *an object oriented formal concept* if $X = Y^\diamond$ and $Y = X^\Box$. The set of objects X is called the extension of the concept (X, Y), and the set of the properties Y is called the intension of the concept (X, Y).

From now on, based on the above facts about the object-oriented concepts studied by Yao, we are trying to study a new type derived from a soft concept based on soft-sets by using two operators defined in Definition 3.1.

We assume that a soft set (F, A) is *pure* [5], that is, $\cup_{a \in A} F(a) = U$, $\cap_{a \in A} F(a) = \emptyset$, $F(a) \neq \emptyset$ and $F(a) \neq U$ for each $a \in A$.

Definition 3.8. Let (U, A, F) be a soft context and $X \in P(U)$. Then X is called an *object oriented soft concept* (simply, *m-concept*) in (U, A, F) if $\mathfrak{F}(X) = \mathbb{F}\overleftarrow{\mathbb{F}}(X) = X$. The set of all m-concepts is denoted by $m(U, A, F)$.

Let (U, A, I) be a formal context in formal concept analysis, where U is a finite nonempty set of objects, A is a finite nonempty set of attributes and I is a binary relation between U and A. Naturally, we can define a soft set $F_I : A \to P(U)$ as follows $F_I(a) = \{x \in U : (x, a) \in I\}$. Then (U, A, F_I) is *the associated soft context* induced by a formal context (U, A, I) (See Remark 3.3 in [6]).

Lemma 3.9. Let (U, A, I) be a formal context. Then for the associated soft context (U, A, F_I) induced by a formal context (U, A, I),

(1) $xI = \{a \in A | xIa\} = \{a \in A | x \in F_I(a)\}$ for $x \in U$.

(2) $Ia = \{x \in U | xIa\} = F_I(a)$ for $a \in A$.

Theorem 3.10. *Let (U, A, I) be a formal context. Then for the associated soft context (U, A, F_I) induced by a formal context (U, A, I),*

(1) $X^\Box = \overleftarrow{\mathbb{F}}_I(X)$;

(2) $Y^\Diamond = \mathbb{F}_I(Y)$;

(3) moreover, for an object oriented formal concept (X, Y), X is an m-concept in the associated soft context (U, A, F_I) induced by (U, A, I).

Proof. Let $X \subseteq U$ and $Y \subseteq A$.

(1) $$\begin{aligned} X^\Box &= \{a \in A | \forall x \in U(xIa \Rightarrow x \in X)\} \\ &= \{a \in A | Ia \subseteq X\} \\ &= \{a \in A | F_I(a) \subseteq X\} \\ &= \overleftarrow{\mathbb{F}}_I(X). \end{aligned}$$

(2) $$\begin{aligned} Y^\Diamond &= \{x \in U | \exists a \in A(xIa \wedge a \in Y)\} \\ &= \{x \in U | xI \cap Y \neq \emptyset\} \\ &= \cup_{a \in Y} Ia \\ &= \cup_{a \in Y} F_I(a) \\ &= \mathbb{F}_I(Y). \end{aligned}$$

(3) For an object oriented formal concept (X, Y), from $X = Y^\Diamond = \mathbb{F}_I(Y)$ and $Y = X^\Box = \overleftarrow{\mathbb{F}}_I(X)$, it follows that $X = \mathbb{F}_I(\overleftarrow{\mathbb{F}}_I(X)) = \mathfrak{F}_I(X)$, and so X is an m-concept in the associated soft context (U, A, F_I) induced by (U, A, I).

□

For a soft context (U, A, F), we can define a binary relation $I_F \subseteq U \times A$ as follows $(x, a) \in I_F \Leftrightarrow x \in F(a)$. Then obviously, (U, A, I_F) is *the associated formal context* induced by a soft context (U, A, F) (See Remark 3.3 in [6]).

Theorem 3.11. *For an m-concept X in a soft context (U, A, F), let $Y = \overleftarrow{\mathbb{F}}(X)$. Then*

(1) $X^\Box = Y$; (2) $Y^\Diamond = X$;

(3) the pair (X, Y) is an object oriented formal concept in the associated formal context (U, V, I_F).

Proof. First, from $x \in F(a) \Leftrightarrow (x, a) \in I_F \Leftrightarrow x \in F_{I_F}$, it is obviously that (U, A, I_F) is the associated formal context of (U, A, F).

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(1) $X^{\Box} = \{a \in A | I_F a \subseteq X\} = \{a \in A | F_{I_F}(a) \subseteq X\} = \{a \in A | F(a) \subseteq X\} = \overleftarrow{\mathbb{F}}(X) = Y$.

(2) $Y^{\Diamond} = \cup_{a \in Y} I_F a = \cup_{a \in Y} F_{I_F}(a) = \cup_{a \in Y} F(a) = \mathbb{F}(Y) = \mathbb{F}(\overleftarrow{\mathbb{F}}(X)) = X$.

(3) By (1) and (2), the pair $(X, Y) = (X, \overleftarrow{\mathbb{F}}(X))$ is an object oriented formal concept in the associated formal context (U, V, I_F).

□

For this reason, an m-concept is also called an object oriented soft concept.

Remark 3.12. In a soft context (U, A, F), the notion of m-soft concepts is independent of the notion of soft concepts to each other, because two notions are induced by two different operations as the following:

For each $X \in P(U)$ and $B \in P(A)$,
(1) $\mathbf{F}^+ : P(A) \to P(U)$ is a mapping defined as $\mathbf{F}^+(B) = \cap_{b \in B} F(b)$;
(2) $\mathbf{F}^- : P(U) \to P(A)$ is a mapping defined as $\mathbf{F}^-(X) = \{a \in A : X \subseteq F(a)\}$;
(3) $\Psi : P(U) \to P(U)$ is an operation defined as $\Psi(X) = \mathbf{F}^+\mathbf{F}^-(X)$.
(4) X is a soft concept [6] if $\Psi(X) = \mathbf{F}^+\mathbf{F}^-(X) = X$.

(1) $\mathbb{F} : P(A) \to P(U)$ is a mapping defined by $\mathbb{F}(B) = \cup_{b \in B} F(b)$.
(2) $\overleftarrow{\mathbb{F}} : P(U) \to P(A)$ is a mapping defined by $\overleftarrow{\mathbb{F}}(X) = \{a \in A : F(a) \subseteq X\}$.
(3) $\mathfrak{F} : P(U) \to P(U)$ is an operation defined by $\mathfrak{F}(X) = \mathbb{F}\overleftarrow{\mathbb{F}}(X)$.
(4) X is an m-concept if $\mathfrak{F}(X) = \mathbb{F}\overleftarrow{\mathbb{F}}(X) = X$.

Example 3.13. Let $U = \{1, 2, 3, 4, 5\}$ and $A = \{a, b, c, d, e\}$. Consider a soft context (U, A, F) where a set-valued mapping $F : A \to P(U)$ is defined by

$$F(a) = \{1, 2, 4\}; \ \ F(b) = \{2, 4, 5\};$$

$$F(c) = \{2, 4\}; \ \ F(d) = \{1, 3\}; \ \ F(e) = \{1, 5\}.$$

Then
$\mathfrak{F}(\{1, 3, 5\}) = \mathbb{F}\overleftarrow{\mathbb{F}}(\{1, 3, 5\}) = \mathbb{F}(\{d, e\}) = \{1, 3, 5\}$;
$\Psi(\{1, 3, 5\}) = \mathbf{F}^+\mathbf{F}^-(\{1, 3, 5\}) = \mathbf{F}^+(\emptyset) \neq \{1, 3, 5\}$,
So, $\{1, 3, 5\}$ is an m-concept but not a soft concept.

And
$\mathfrak{F}(\{1\}) = \mathbb{F}\overleftarrow{\mathbb{F}}(\{1\}) = \mathbb{F}(\emptyset) \neq \{1\}$; $\Psi(\{1\}) = \mathbf{F}^+\mathbf{F}^-(\{1\}) = \mathbf{F}^+(\{a, d, e\}) = \{1\}$,
So, $\{1\}$ is soft concept but not an m-concept.

Theorem 3.14. *Let (U, A, F) be a soft context. Then we have:*

(1) $\mathfrak{F}(\emptyset) = \emptyset$.

(2) $\mathfrak{F}(X)$ is an m-concept.

(3) For $B \subseteq A$, $\mathbb{F}(B)$ is an m-concept.

(4) For $a \in A$, $F(a)$ is an m-concept.

(5) X is an m-concept if and only if there is some $B \subseteq A$ such that $X = \mathbb{F}(B)$.

Proof. (1) Obvious.

(2) It follows from (4) of Theorem 3.2.

(3) By Theorem 3.2, $\mathfrak{F}(\mathbb{F}(B)) = (\mathbb{F}\overleftarrow{\mathbb{F}})(\mathbb{F}(B)) = (\mathbb{F}\overleftarrow{\mathbb{F}}\mathbb{F})(B) = \mathbb{F}(B)$, so $\mathbb{F}(B)$ is an m-concept.

(4) Since $\mathbb{F}(\{a\}) = F(a)$, it is obvious.

(5) Let X be an m-concept and $X \neq \emptyset$. Put $\overleftarrow{\mathbb{F}}(X) = B$. Then B is a nonempty subset of A, and since X is an m-concept, we have that $\mathbb{F}(B) = \mathbb{F}\overleftarrow{\mathbb{F}}(X) = \mathfrak{F}(X) = X$. For the proof of the another part, for any nonempty subset X of U, suppose that there exists $B \subseteq A$ such that $\mathbb{F}(B) = X$. Then $\mathfrak{F}(X) = \mathbb{F}\overleftarrow{\mathbb{F}}(X) = \mathbb{F}\overleftarrow{\mathbb{F}}\mathbb{F}(B) = \mathbb{F}(B) = X$ and so X is an m-concept.

□

Theorem 3.15. *Let (U, A, F) be a soft context and $\mathbf{Im}(\mathbb{F}) = \{\mathbb{F}(C) \mid \mathbb{F} : P(A) \to P(U), \ C \in P(A)\}$. Then*

(1) $\mathbf{Im}(\mathbb{F}) = m(U, A, F)$:

(2) For $C_1, \cdots, C_n \subseteq A$, $\mathbb{F}(C_1) \cup \mathbb{F}(C_2) \cup \cdots, \mathbb{F}(C_n) \in \mathbf{Im}(\mathbb{F})$.

Proof. (1) It is obtained from (5) of Theorem 3.14.

(2) By (3) of Theorem 3.2, $\mathbb{F}(C_1) \cup \mathbb{F}(C_2) \cup \cdots, \mathbb{F}(C_n) = \mathbb{F}(C_1 \cup C_2 \cup \cdots, \cup C_n) \in \mathbf{Im}(\mathbb{F})$.

□

Theorem 3.16. *Let (U, A, F) be a soft context and $\mathcal{F} = \{F(a) \mid \ a \in A\}$. Then*

(1) $\mathcal{F} \subseteq m(U, A, F)$:

(2) For each $X \in m(U, A, F)$, there exist $B_1, B_2, \cdots, B_n$ in $\mathcal{F}$ satisfying $X = \cup B_i$, $i = 1, 2, \cdots, n$.

Proof. (1) Obviously it follows from (1) of Theorem 3.15.

(2) Let $X \in m(U, A, F)$. Then there is $B \in P(A)$ such that $X = \mathbb{F}(B)$ by (5) of Theorem 3.14. From $\{\{b\} \mid b \in B\} \subseteq \mathcal{F}$, it follows that $X = \mathbb{F}(B) = \cup_{b \in B} \mathbb{F}(\{b\})$. So, the statement (2) is obtained. □

By using Theorem 3.15 and 3.16, we can easily construct the set $m(U, A, F)$ of all m-concepts in a given soft context:

Example 3.17. Let $U = \{1, 2, 3, 4, 5\}$ and $A = \{a, b, c, d, e\}$. Consider a soft context (U, A, F) where the set-valued mapping $F : A \to P(U)$ is defined as follows:

$$F(a) = \{1, 2, 4\}; \ F(b) = \{1, 2, 4, 5\};$$

$F(c) = \{2,4\};\ F(d) = \{1,3\};\ F(e) = \{1,5\}.$

Then,

m(U,A,F)=
$\{\emptyset, \{1,3\}, \{1,5\}, \{2,4\}, \{1,2,4\}, \{1,3,5\}, \{1,2,3,4\}, \{1,2,4,5\}, U\}.$

4. CONCLUSION

We introduced the notion of m-concept in a soft context induced by a soft set. Then we showed that the class of all the m-concepts is a image of some subset of attributes on a given soft set. In the next research, we will study the special properties of the m-concept related with the topological structure, and characterizations for m-concepts by using a nonempty finite set of attributes on a given soft set.

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