International Journal of Engineering Research and Technology. ISSN 0974-3154, Volume 12, Number 12 (2019), pp. 2510-2515

Study of Multidimensional and Nonlinear Control Systems, Built in the Class of "Hyperbolic Umbilic"

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Abstract:

This paper proposes a method of constructing a control system for deterministic chaotic modes of nonlinear objects in a class of hyperbolic umbilic catastrophe from the catastrophe theory for multidimensional and nonlinear systems. It is shown that an increase in the potential of robust stability is the primary factor in the protection, ensuring the system from the deterministic chaos mode with an emergence of "strange attractors". A study of control systems with a high potential for robust stability is implemented by the gradient-velocity method of Vector Lyapunov Functions.

Modern control problems are characterized by increasing complexity and nonlinearity of control objects, stability requirements of control systems in conditions of uncertainty and incomplete information. In recent years, the possibility of chaotic dynamics has been discovered in a large number of non-linear systems. Currently, it is generally accepted that the real objects of control are non-linear and one of the basic properties of nonlinear dynamical systems is an attainment of the deterministic chaos mode with a formation of "strange attractors".

Keywords - Control, Robust Stability, Nonlinear System, Strange Attractor, Hyperbolic Umbilic.

I. INTRODUCTION

As it has been already emphasized, a property of deterministic chaos, bifurcation, self-organization is defined by a property of stability and is associated with the system response to disturbances of various types [1-10]. Therefore, we introduce a "standard state" $X_{S1},...,X_{Si},...,X_{Sn}$, where $\{X_{Si}\}$ represents a set of state variables, continuously probed by internal fluctuations or external disturbances. If we deal with a completely unperturbed system, i.e if the system has no external disturbance and control, then it is true that $X_S = (X_{S1},...,X_{Sn})$ will refer to the time-independent stationary solution. This result means that the system has no deterministic chaos and complex behavior types [11-13]. Thus, the occurrence of complex behavior and deterministic chaos can be considered as a transition from $X_S = (X_{1S},...,X_{nS})$ to the solutions of the new type.

Let us now consider what should be a formalism that allows to find out the stability of the studied standard state. The main difficulty in the theory of stability is related to the large number of variables that describe this problem [14-16].

II. RESULTS AND DISCUSSION

A nonlinear stationary control system is described by the equation of state:

$$\dot{X} = f(X) + Bu,\ X \in R^n,\ u \in R^m \tag{1}$$

where, $f(\bullet)$ is a vector - a function of dimension $n\left(f(\bullet) \in R^n\right)$, u (t) is a vector - function of dimension m, defined in the form of three-parameter structurally stable mappings.

$$u_i = -X_i^3 - X_{i+1}^3 - k_{i,i+1}^o X_i X_{i+1} + k_i X_i + k_{i+1} X_{i+1},\quad i = 1,...,n-1 \tag{2}$$

A control matrix is defined in the diagonal form for comfort.

$$B = \begin{Vmatrix} b_{11} & 0 & 0, & ..., & 0 \\ 0 & b_{22} & 0, & ..., & 0 \\ 0 & 0 & b_{33}, & ..., & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0, & ..., & b_{mm} \end{Vmatrix}$$

A nonlinear stationary object is described by the equation:

$$\dot{X} = f(X) \tag{3}$$

The state of equation the control object in the expanded form can be written as:

$$\begin{aligned} \dot{X}_1 &= f_1(X_1,...,X_n) \\ \dot{X}_2 &= f_2(X_1,...,X_n) \\ &\dots\dots\dots\dots\dots\dots \\ \dot{X}_n &= f_n(X_1,...,X_n) \end{aligned} \tag{4}$$

All functions in the right-hand side of the state equation (4) are given in the canonical form using a number of theorems from the theory of catastrophes. Change in X in the process of control $(u(t)=0)$ and disturbances $(z(t)=0)$ to zero, i.e. free movement of the system can be written in the following form:

$$\frac{dX}{dt}=f(X) \quad (5)$$

Here $f(\bullet)$ - a vector-function operating in the space, which defines a vector $X=(X_1,...,X_n)^T$. This vector-function is usually nonlinear.

Derived above standard state X_S is a particular solution of equation (5). Consequently:

$$\frac{dX_S}{dt}=f(X_S)$$

Using this representation from (5) we can obtain equation for x:

$$\frac{dx}{dt}=f(X_S+x)-f(X_S)$$

Naturally, this equation can be expanded, i.e. its right-hand side near any standard state X_S. If a f has the form of a polynomial in terms of X, then it is always possible that leads to a finite number of terms. However, in more complex cases f may depend on X in any other way. In this case, we assume that: 1) f continues to be expanded in a power series of X and 2) the expansion can be truncated at power of a finite order. By virtue of the latter assumption, usually we have to be sometimes limited to the study of infinitesimal stability, i.e. study of the system response to small perturbations, such that $|x|/|X_S|<<1$. This limitation is often of minor importance, since the infinitesimal stability gives a necessary condition for instability in the sense that if X_S is unstable with respect to small X, then it will be unstable with respect to any X.

Formally, the above-described expansion in state space $x(t)\in R^n$ can be represented by the equation:

$$\frac{dx}{dt}=\left(\frac{\partial f}{\partial X}\right)_{x_S}x+\frac{1}{2}\left(\frac{\partial^2 f}{\partial X\partial X}\right)_{x_S}xx+\frac{1}{6}\left(\frac{\partial^3 f}{\partial X\partial X\partial X}\right)_{x_S}xxx+\cdots \quad (6)$$

Equations (6) corresponds to the description of the free movement of system, comprising deviations x relative to the standard state X_s.

The state equations of the control object in the expanded form can be represented in the following form:

$$\frac{dx_i}{dt}=\sum_{j=1}^{n}\frac{\partial f_i(X)}{\partial X_j}\bigg|_{X_S}x_j+\frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{\partial^2 f_i(X)}{\partial X_j\partial X_k}\bigg|_{X_S}x_jx_k+\frac{1}{6}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{m=1}^{n}\frac{\partial^3 f_i(X)}{\partial X_j\partial X_k\partial X_m}\bigg|_{X_S}\cdot$$
$$\cdot x_jx_kx_m+\frac{1}{24}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{m=1}^{n}\sum_{l=1}^{n}\frac{\partial^4 f(X)}{\partial X_j\partial X_k\partial X_m\partial X_l}\bigg|_{X_S}x_jx_kx_mx_l+,\cdots \qquad i=1,...,n \quad (7)$$

Nonlinear stationary control object with m inputs and n outputs, using a smooth change of variables in the canonical form can be written as:

$$\frac{dx_i}{dt}=\frac{\partial f_i(X)}{\partial X_1}\bigg|_{X_S}x_1+\frac{\partial f_i(X)}{\partial X_2}\bigg|_{X_S}x_2+,\cdots,-x_i^3-x_{i+1}^3-\frac{\partial^2 f_i(X)}{\partial X_i\partial X_{i+1}}\bigg|_{X_S}x_ix_{i+1}+$$
$$+\frac{\partial_i(X)}{\partial X_{i+1}}\bigg|_{X_S}x_{i+1},\cdots,\frac{\partial f_i(X)}{\partial X_{n-1}}\bigg|_{X_S}x_{n-1}+\frac{\partial f_i(X)}{\partial X_n}\bigg|_{X_S}x_n+\cdots \quad i=1,...,n \quad (8)$$

where,

$$\frac{\partial f_i(X)}{\partial X_1}\bigg|_{X_S}=a_{i1},\ \frac{\partial f_i(X)}{\partial X_2}\bigg|_{X_S}=a_{i2},\ \frac{\partial f_i(X)}{\partial X_i}\bigg|_{X_S}=a_{ii},$$
$$\frac{\partial f_i(X)}{\partial X_{i+1}}\bigg|_{X_S}=a_{i,i+1},\cdots,\ \frac{\partial f_i(X)}{\partial X_{n-1}}\bigg|_{X_S}=a_{i,n-1},\ \frac{\partial f_i(X)}{\partial X_n}\bigg|_{X_S}=a_{in},$$
$$\frac{\partial f_i(X)}{\partial X_i\partial X_{i+1}}\bigg|_{X_S}=k_{i,k+1}^{o}$$

Nonlinear stationary control system (1) subject to the control procedure (2) and a mathematical model of control object (8) in the expanded form can be presented as the following system of equations:

$$\begin{cases} \dot{x}_1 = -b_{11}x_1^3 - b_{11}x_2^3 - b_{11}k_{12}x_1x_2 + (a_{11} + b_{11}k_1)x_1 + (a_{12} + b_{11}k_2)x_2 + a_{13}x_1 +,...,+ a_{1n}x_n \\ \dot{x}_2 = -b_{22}x_1^3 - b_{22}x_2^3 - b_{22}k_{12}x_1x_2 + (a_{21} + b_{22}k_1)x_1 + (a_{22} + b_{22}k_2)x_2 + a_{23}x_3 +,...,+ a_{2n}x_n \\ \dot{x}_3 = -b_{33}x_3^3 - b_{33}x_4^3 - b_{33}k_{34}x_3x_4 + a_{32}x_2 + (a_{33} + b_{33}k_3)x_3 + (a_{34} + b_{33}k_4)x_4 + a_{35}x_5 +,...,+ a_{3n}x_n \\ \dot{x}_4 = -b_{44}x_3^3 - b_{44}x_4^3 - b_{44}k_{34}x_3x_4 + a_{41}x_1 + a_{42}x_2 + (a_{43} + b_{44}k_3)x_3 + (a_{44} + b_{44}k_4)x_4 + \\ + a_{45}x_5 +,...,+ a_{4n}x_n \\ \cdots \quad \cdots \quad \cdots \quad \cdots\cdots \quad \cdots \quad \cdots \quad \cdots \\ \dot{x}_{n-1} = -b_{n-1,n-1}x_{n-1}^3 - b_{n-1,n-1}x_n^3 - b_{n-1,n-1}k_{n-1,n}x_{n-1}x_n + a_{n1}x_1 + a_{n2}x_2 +,...,+ (a_{n-1,n-1} + b_{n-1,n-1}k_{n-1})x_{n-1} + \\ + (a_{n-1,n} + b_{n-1,n-1}k_n)x_n \\ \dot{x}_n = -b_{n,n}x_n^3 - b_{n,n}x_{n-1}^3 - b_{n,n}k_{n-1,n}x_{n-1}x_n + a_{n1}x_1 + a_{n2}x_2 +,...,+ (a_{n,n-1} + b_{n,n}k_{n-1})x_{n-1} + (a_{n,n} + b_{n,n}k_n)x_n \end{cases} \tag{9}$$

where $k_{i,i+1}^o + k_{i,i+1}^c = k_{i,i+1},\ i = 1,...,n-1$

Obtained state of the system (9) will be determined by solving equations:

$$\begin{cases} -b_{11}x_{1s}^3 - b_{11}x_{2s}^3 - b_{11}k_{12}x_{1s}x_{2s} + (a_{11} + b_{11}k_1)x_{1s} + (a_{12} + b_{11}k_2)x_{2s} + a_{13}x_{3s} +,...,+ a_{1n}x_{ns} = 0 \\ -b_{22}x_{1s}^3 - b_{22}x_{2s}^3 - b_{22}k_{12}x_{1s}x_{2s} + (a_{21} + b_{22}k_1)x_{1s} + (a_{22} + b_{22}k_2)x_{2s} + a_{23}x_{3s} +,...,+ a_{2n}x_{ns} = 0 \\ \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \\ a_{n1}x_{1s} + a_{n2}x_{2s} +,...,- b_{nm}x_{ns}^3 - b_{nm}x_{n-1,s}^3 - b_{nn}k_{n-1,n}x_{n-1,s}x_{ns} + (a_{n,n-1} + b_{nn}k_{n-1})x_{n-1,s} + (a_{nn} + b_{nn}k_n)x_{n,s} = 0 \end{cases} \tag{10}$$

From the system of equations (10) we can find the stationary states:

$$x_{1s} = 0,\ x_{2s} = 0,...,x_{ns} = 0, \tag{11}$$

Some other stationary states can be determined by solving equations:

$$-b_{ii}x_{is}^2 + a_{ii} + b_{ii}k_i = 0,\ x_{js} = 0 \quad \text{at} \quad i \neq j, i = 1,...,n \tag{12}$$

Equation (12) with negative $k_i + \frac{a_{ii}}{b_{ii}}\ i = 1,...,n,$ ($k_i + \frac{a_{ii}}{b_{ii}}$ <0, $i = 1,...,n$) have imaginary solutions that cannot correspond to any physical possible situation. When $k_i - \frac{a_{ii}}{b_{ii}}$ greater than zero ($k_i - \frac{a_{ii}}{b_{ii}} > 0$), equation (12) admits the following steady states:

$$x_{is} = \sqrt{k_i + \frac{a_{ii}}{b_{ii}}},\ x_{js} = 0 \ \text{at}\ i \neq j, i = 1,...,n; j = 1,...,n \tag{13}$$

and

$$x_{is} = -\sqrt{k_i + \frac{a_{ii}}{b_{ii}}},\ x_{js} = 0 \ \text{at}\ i \neq j, i = 1,...,n; j = 1,...,n \tag{14}$$

International Journal of Engineering Research and Technology. ISSN 0974-3154, Volume 12, Number 12 (2019), pp. 2510-2515
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Stability of the stationary states (11), (13) and (14) of the system (9) is investigated by gradient-velocity method of Vector Lyapunov function [9,10].

1. Consider the stability of stationary state (11). We find (9) antigradient vector components of the vector function at $V(x) = (V_1(x), V_2(x), ..., V_n(x))$:

$$\begin{cases} \dfrac{\partial V_1(x)}{\partial x_1} = b_{11}x_1^3 + \dfrac{1}{2}b_{11}k_{12}x_1x_2 - b_{11}(k_1 + \dfrac{a_{11}}{b_{11}})x_1 \\ \dfrac{\partial V_1(x)}{\partial x_2} = b_{11}x_2^3 + \dfrac{1}{2}b_{11}k_{12}x_1x_2 - b_{11}(k_2 + \dfrac{a_{11}}{b_{11}})x_2 \\ \dfrac{\partial V_1(x)}{\partial x_3} = -a_{13}x_3, \dfrac{\partial V_1(x)}{\partial x_4} = -a_{14}x_4, ..., \dfrac{\partial V_1(x)}{\partial x_n} = -a_{1n}x_n \\ \dfrac{\partial V_2(x)}{\partial x_1} = b_{22}x_1^3 + \dfrac{1}{2}b_{22}k_{12}x_1x_2 - b_{22}(k_1 + \dfrac{a_{21}}{b_{22}})x_1 \\ \dfrac{\partial V_2(x)}{\partial x_2} = b_{22}x_2^3 + \dfrac{1}{2}b_{22}k_{22}x_1x_2 - b_{22}(k_2 + \dfrac{a_{22}}{b_{22}})x_2 \\ \dfrac{\partial V_2(x)}{\partial x_3} = -a_{23}x_3, \dfrac{\partial V_2(x)}{\partial x_4} = -a_{24}x_4, ..., \dfrac{\partial V_2(x)}{\partial x_n} = -a_{2n}x_n \\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots \\ \dfrac{\partial V_n(x)}{\partial x_1} = -a_{n1}x_1, \dfrac{\partial V_n(x)}{\partial x_2} = -a_{n2}x_2, \\ \dfrac{\partial V_n(x)}{\partial x_3} = -a_{n3}x_3, ..., \dfrac{\partial V_n(x)}{\partial x_{n-1}} = b_{nn}x_n^3 + \dfrac{1}{2}b_{nn}k_{n-1,n}x_{n-1}x_n - b_{nn}(k_{n-1} + \dfrac{a_{n,n-1}}{b_{nn}})x_{n-1} \\ \dfrac{\partial V_n(x)}{\partial x_n} = b_{nn}x_n^3 + \dfrac{1}{2}b_{nn}k_{n-1,n}x_{n-1}x_n - b_{nn}(k_n + \dfrac{a_{nn}}{b_{nn}})x_n \end{cases} \tag{15}$$

From (9) we define projections of the velocity vector components in the coordinate system.

Find the total time derivative of the Lyapunov vector-function as the scalar product of the gradient vector (15) to the velocity vector.

$$\left(\frac{dV(x)}{dt}\right) = -b_{11}^2\left[x_1^3 + \frac{1}{2}k_{12}x_1x_2 - \left(k_1 + \frac{a_{11}}{b_{11}}\right)x_1\right]^2 - b_{11}^2\left[x_2^3 + \frac{1}{2}k_{12}x_1x_2 - \left(k_2 + \frac{a_{12}}{b_{11}}\right)x_2\right]^2 -$$
$$- a_{13}^3x_3^2 -, ..., -a_{1n}^2x_n^2 - b_{22}^2\left[x_1^3 + \frac{1}{2}k_{12}x_1x_2 - \left(k_2 + \frac{a_{21}}{b_{22}}\right)x_1\right]^2 - b_{22}^2\left[x_1^3 + \frac{1}{2}k_{12}x_1x_2 - \left(k_2 + \frac{a_{22}}{b_{22}}\right)x_2\right]^2 -$$
$$- a_{23}^2x_3^2 -, ..., -a_{2n}^2x_n^2, ..., -a_{n1}^2x_1^2 - a_{n2}^2x_2^2 -, ..., -b_{nn}^2\left[x_{n-1}^3 + \frac{1}{2}k_{n-1,n}x_{n-1}x_n - \left(k_{n-1} - \frac{a_{n,n-1}}{b_{nn}}\right)x_{n-1}\right]^2 -$$
$$- b_{nn}^2\left[x_n^3 + \frac{1}{2}k_{n-1,n}x_{n-1}x_n - \left(k_n - \frac{a_{nn}}{b_{nn}}\right)x_n\right]^2 \tag{16}$$

From (16) it is clear that the total time derivative of the Liapunov function vector is a negative function, therefore we meet a sufficient condition for the asymptotic stability of the system.

By gradient components (15) we construct a vector of Lyapunov function in the scalar form.

Condition of positive and negative states of Lyapunov function V (x) is not obvious from (17), so we can use the lemma of Morse from the theory of catastrophes.

$$V_1(x) = \frac{1}{4}b_{11}x_1^4 + \frac{1}{4}b_{11}k_{12}x_1^2x_2 + \frac{1}{2}b_{11}\left(k_1 + \frac{a_{11}}{b_{11}}\right)x_1^2 + \frac{1}{4}b_{11}x_2^4 + \frac{1}{4}b_{11}k_{12}x_1^2x_2^2 -$$
$$- \frac{1}{2}b_{11}\left(k_2 + \frac{a_{12}}{b_{11}}\right)x_2 - \frac{1}{2}a_{13}x_3^2 - ... - \frac{1}{2}a_{1n}x_n^2 + \frac{1}{4}b_{22}x_1 + \frac{1}{4}b_{22}k_{12}x_1^2x_2 -$$
$$- \frac{1}{2}\left(k_1 + \frac{a_{21}}{b_{11}}\right)x_1^2 + \frac{1}{4}b_{22}x_2^4 + \frac{1}{4}b_{22}k_{12}x_1x_2^2 - \frac{1}{2}\left(k_2 + \frac{a_{22}}{b_{22}}\right)x_2^2 - \frac{1}{2}a_{23}x_3^2 - ... -$$
$$- \frac{1}{2}a_{2n}x_n^2 - ... - \frac{1}{2}a_{n1}x_1^2 - \frac{1}{2}a_{n2}x_2^2 - ... - \frac{1}{2}a_{n,n-1}x_{n-2}^2 + \frac{1}{4}b_{nn}x_{n-1}^4 +$$
$$+ \frac{1}{4}b_{nn}k_{n-1,n}x_{n-1}^2x_n - \frac{1}{2}b_{nn}\left(k_{n-1} + \frac{a_{n,n-1}}{b_{nn}}\right)x_{n-1}^2 + \frac{1}{4}b_{nn}x_n^4 + \frac{1}{4}b_{nn}k_{n-1,n}x_{n-1}x_n^2 -$$
$$- \frac{1}{2}b_{nn}\left(k_2 + \frac{a_{nn}}{b_{nn}}\right)x_n^2 \tag{17}$$

By Lemma Morse, Lyapunov function (17) in the vicinity of the stationary state (11) can locally be represented as a quadratic form considering the state equation (9):

$$V_{1j}(x) \approx [(b_{11} + b_{22})k_1 + a_{11} + a_{21} + ... + a_{n1}]x_1^2 - [(b_{11} + b_{22})k_2 + a_{12} + a_{22} + ... + a_{n2}]x_2^2 -$$
$$- ... - [(b_{n-1,n-1} + b_{nn})k_n + a_{1n} + a_{2n} + ... + a_{nn}]x_n^2 \tag{18}$$

Conditions of the positive definiteness in the quadratic form (18) (stability of stationary state (11) determined by the system of inequalities):

$$\begin{cases} (b_{11} + b_{22})k_1 + a_{11} + a_{21} + ... + a_{n1} < 0 \\ (b_{11} + b_{22})k_2 + a_{12} + a_{22} + ... + a_{n2} < 0 \\ \cdots\ \cdots\cdots\cdots \\ (b_{n-1,n-1} + b_{nn})k_n + a_{1n} + a_{2n} + ... + a_{nn} < 0 \end{cases} \tag{19}$$

Thus, stability region of the steady state (11) will be determined by compliance with the system of inequalities (19), built relative to the undefined parameters of the control object and selected parameters of the control system.

2. We investigate the stability of the stationary states (13) and (14). The equation of state (9) is represented in the deviations relative to the stationary state (13) and (14), using the known formalism. We compute the values [17,18] of the derivatives in the right-hand side of the state equations (9) at the stationary point of states (13).

Using the given formalism [12,13] in the state equation (9) in deviations relative to a stationary state (13) or (14) can be recorded in the following form:

$$
\begin{cases}
\dot{x} = -b_{11}x_1^3 - b_{11}x_2^3 - b_{11}k_{12}x_1x_2 - 3b\sqrt{k_1 + \frac{a_{11}}{b_{11}}x_1^2} - 3b_{11}\sqrt{k_2 + \frac{a_{12}}{b_{11}}x_2^2} - \\
-2b_{11}\left(k_1 + \frac{a_{11}}{b_{11}}\right)x_1 - 2b_{11}\left(k_2 + \frac{a_{12}}{b_{11}}\right)x_2 + a_{13}x_3 + \ldots + a_{1n}x_n \\
\dot{x} = -b_{22}x_1^3 - b_{22}x_2^3 - b_{22}k_{12}x_1x_2 - 3b_{22}\sqrt{k_1 + \frac{a_{21}}{b_{11}}x_1^2} - 3b_{22}\sqrt{k_2 + \frac{a_{22}}{b_{22}}x_2^2} - \\
-2b_{22}\left(k_1 + \frac{a_{21}}{b_{22}}\right)x_1 - 2b_{22}\left(k_2 + \frac{a_{22}}{b_{22}}\right)x_2 + a_{23}x_3 + \ldots + a_{2n}x_n \\
\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\
\dot{x}_n = a_{n1}x_1 + \ldots + a_{n2}x_2 + \ldots + b_{nn}x_{n-1}^3 - b_{nn}x_n^3 - b_{nn}k_{n-1}x_{n-1}x_n - \\
-3b_{nn}\sqrt{k_{n-1} + \frac{a_{n,n-1}}{b_{n-1,n}}x_{n-1}^2} - 3b_{nn}\sqrt{k_n + \frac{a_{nn}}{b_{nn}}x_n^2} - 2b_{nn}\left(k_{n-1} + \frac{a_{n,n-1}}{b_{nn}}\right)x_{n-1} - 2b_{nn}\left(k_n + \frac{a_{nn}}{b_{nn}}\right)x_n
\end{cases}
\tag{20}
$$

The control system of deterministic chaotic modes of nonlinear objects with a single input and a single output in the class of hyperbolic umbilic catastrophe.

Find the total time derivative of the Lyapunov vector-function as the dot product of the velocity vector and the gradient vector:

$$
\begin{aligned}
\frac{dV(x)}{dt} = &-b_{11}^2\left[x_1^3 + \frac{1}{2}k_{12}x_1x_2 + 3\sqrt{k_1 + \frac{a_{11}}{b_{11}}x_1^2} + 2\left(k_1 + \frac{a_{11}}{b_{11}}\right)x_1\right]^2 - \\
&-b_{11}^2\left[x_2^3 + \frac{1}{2}k_{12}x_1x_2 + 3\sqrt{k_2 + \frac{a_{12}}{b_{11}}x_2^2} + 2\left(k_2 + \frac{a_{12}}{b_{11}}\right)x_2\right]^2 - a_{13}^2x_3^2, \ldots, -a_{1n}^2x_n^2 \\
&-b_{22}^2\left[x_1^3 + \frac{1}{2}k_{12}x_1x_2 + 3\sqrt{k_1 + \frac{a_{21}}{b_{22}}x_1^2} + 2\left(k_1 + \frac{a_{21}}{b_{22}}\right)x_1\right]^2 - \\
&-b_{22}^2\left[x_2^3 + \frac{1}{2}k_{12}x_1x_2 + 3\sqrt{k_2 + \frac{a_{22}}{b_{22}}x_2^2}\right] - a_{23}^2x_3^2 -, \ldots, -a_{2n}^2x_n^2 - \\
&\ldots \ldots \ldots \ldots \ldots \\
&-a_{n1}^2x_1^2 - a_{n2}^2x_2^2 -, \ldots, - \\
&-b_{nn}^2\left[x_{n-1}^3 + \frac{1}{2}k_{n-1,n}x_{n-1}x_n + 3\sqrt{k_{n-1} - \frac{a_{n,n-1}}{b_{nn}}x_{n-1}^2} + 2\left(k_{n-1} - \frac{a_{n,n-1}}{b_{nn}}\right)x_{n-1}\right]^2 - \\
&-b_{nn}^2\left[x_n^3 + \frac{1}{2}k_{n-1,n}x_{n-1}x_n + 3\sqrt{k_n - \frac{a_{nn}}{b_{nn}}x_n^2} + 2\left(k_n + \frac{a_{nn}}{b_{nn}}\right)x_n\right]^2,
\end{aligned}
\tag{21}
$$

From (21) it is clear that the total time derivative of the Liapunov vector-function is a negative function, therefore, a sufficient condition is met for asymptotic system stability (20).

Using components of Lyapunov vector-function we construct the Lyapunov vector-function in the scalar form:

$$
\begin{aligned}
V(x) = &\frac{1}{4}b_{11}x_1^4 + \frac{1}{4}b_{11}k_{12}x_1^2x_2 + b_{11}\sqrt{k_1 + \frac{a_{11}}{b_{11}}x_1^3} + b_{11}\left(k_1 + \frac{a_{11}}{b_{11}}\right)x_1^2 + \\
&+\frac{1}{4}b_{22}x_2^4 + \frac{1}{4}b_{22}k_{12}x_1^2x_2 + b_{22}\sqrt{k_2 + \frac{a_{12}}{b_{22}}x_2^2} + b_{11}\left(k_2 + \frac{a_2}{b_{22}}\right)x_2^2 - \frac{1}{2}a_{13}x_3^2 - \ldots - a_{1n}x_n^2 +, \ldots, + \\
V(x) = &-\frac{1}{2}a_{n1}x_1^2 - \frac{1}{2}a_{n2}x_2^2 - \ldots + \frac{1}{4}b_{nn}x_{n-1}^4 + \frac{1}{4}b_{nn}k_{n-1}x_{n-1}^2x_n + b_{nn}\sqrt{k_{n-1} + \frac{a_{nn-1}}{b_{nn}}x_{n-1}^3} + \\
&+b_{nn}\left(k_{n-1} + \frac{a_{n,n-1}}{b_{nn}}\right)x_{n-1}^2 + \frac{1}{4}b_{nn}x_n^4 + \frac{1}{4}b_{nn}k_{n-1,n}x_{n-1}x_n^2 + b_{nn}\sqrt{k_n + \frac{a_{nn}}{b_{nn}}x_n^3} + b_{nn}\left(k_n + \frac{a_{nn}}{b_{nn}}\right)x_n^2
\end{aligned}
\tag{22}
$$

From the expressions (22) positive or negative definition of Lyapunov function cannot be defined, so we use the fundamental theorem of the theory of catastrophes - Morse Lemma [19-21]. By Lemma Morse, Lyapunov function (22) in the vicinity of the stationary state (13) and (14) can be locally presented in the quadratic form considering the state equation (20) to deviations with respect to the stationary state (13) or (14):

$$
\begin{aligned}
V(x) = &\left[(b_{11} + b_{22})k_1 + a_{11} + a_{21} - \frac{1}{2}(a_{31} + a_{41} +, \ldots, + a_n)\right]x_1^2 + \\
&+\left[(b_{11} + b_{22})k_2 + a_{12} + a_{22} - \frac{1}{2}(a_{32} + a_{42} +, \ldots, + a_{n2})\right]x_2^2 + \\
&+\left[(b_{33} + b_{44})k_3 + a_{33} + a_{43} - \frac{1}{2}(a_{13} + a_{23} +, \ldots, + a_{n3})\right]x_3^2 +, \ldots, + \\
&+\left[(b_{33} + b_{44})k_4 + a_{44} + a_{34} - \frac{1}{2}(a_{14} + a_{24} +, \ldots, + a_{n4})\right]x_4^2 +, \ldots, + \\
&+\left[(b_{n-1,n-1} + b_{nn})k_{n-1} + a_{n-1,n-1} + a_{n,n-1} - \frac{1}{2}(a_{1,n-1} + a_{2,n-1} + a_{3,n-1} +, \ldots, + a_{n,n-1})\right]x_{n-1}^2 + \\
&+\left[(b_{n-1,n-1} + b_{nn})k_n + a_{n-1,n} + a_{nn} - \frac{1}{2}(a_{1n} + a_{2n} + a_{3n} +, \ldots, + a_{n-2,n})\right]x_n^2,
\end{aligned}
\tag{23}
$$

Stability conditions of stationary state (13) or (14) are determined by the positive definiteness of the quadratic form (23), i.e. by the system of inequalities:

$$
\begin{cases}
(b_{11} + b_{22})k_1 + a_{11} + a_{21} > \frac{1}{2}(a_{31} + a_{41} +, \ldots, + a_{n1}) \\
(b_{11} + b_{22})k_2 + a_{12} + a_{22} > \frac{1}{2}(a_{32} + a_{42} +, \ldots, + a_{n2}) \\
(b_{33} + b_{44})k_3 + a_{33} + a_{43} > \frac{1}{2}(a_{13} + a_{23} +, \ldots, + a_{n3}) \\
(b_{33} + b_{44})k_4 + a_{34} + a_{44} > \frac{1}{2}(a_{14} + a_{24} +, \ldots, + a_{n4}) \\
\ldots \ldots \ldots \ldots \ldots \\
(b_{n-1,n-1} + b_{nn})k_{n-1} + a_{m-1,m-1} + a_{n,n-1} > \frac{1}{2}(a_{1,n-1} + a_{2,n-1} +, \ldots, + a_{n,n-1}) \\
(b_{n-1,n-1} + b_{nn})k_n + a_{n-1,n} + a_{nn} > \frac{1}{2}(a_{1n} + a_{2n} +, \ldots, + a_{n-2,n})
\end{cases}
\tag{24}
$$

Stability conditions of stationary state (13) or (14) are determined by the positive definiteness of the quadratic form (23), i.e. by the system of inequalities (24).

Thus, nonlinear MIMO system constructed in the class of three-parameter structurally stable maps will be stable in the unlimited wide range of changes in the undetermined parameters of the control object. The steady state (11) exists and it is stable when changing the undetermined parameters in the region (20), whereas stationary state (13) and (14) exists in case of the loss of stability in the stationary state (11) and they do not exist simultaneously.

Stable stationary states (13) and (14) can be obtained when inequalities (24) are true.

III. CONCLUSIONS

The real control objects are nonlinear, multidimensional and their control systems are designed and operated under conditions of uncertainty. The basic property of nonlinear dynamic control objects is functioning in the deterministic chaos mode with attainment of "strange attractor". The deterministic chaotic modes of control objects can lead to accidents and crisis, and the chaos is shown in the form of loss of stability of the existing stationary state of the system in the conditions of uncertainty.

We propose to solve control problems of deterministic chaotic processes by building control systems with a high potential for robust stability in the class of hyperbolic umbilic catastrophe. The study of control system is perfromed using gradient-velocity method of the Vector Lyapunov function.

The stability region of stationary state in the control system is obtained in the form of a system of inequalities for the simplest uncertain parameters of the closed system. The control system with a high potential for robust stability, built in the class of hyperbolic umbilic catastrophe provides robust stability for any changes in uncertain parameters. Hence, a deterministic chaotic mode is removed from the scenario of development process.

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