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Markov Modelling of Fault-Tolerant Wireless Sensor Networks with Independent Repair

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Abstract

Technological advancements have led to the proliferation of wireless sensor networks (WSNs) in a wide variety of application domains. Many WSN applications are mission-critical, requiring continuous operation. However, WSN sensor nodes are often deployed in unattended and hostile environments making them more susceptible to failures than other systems. Nevertheless, in order to meet application requirements reliably, WSNs require fault tolerance mechanisms. To improve reliability in such systems, we propose a fault-tolerant (FT) sensor node model for such applications. We develop a Markov model of a triplex sensor node repairable system with independent repair facilities for characterizing WSN reliability and Mean Time to Failure (MTTF). Our model focuses on providing repair for faulty sensor nodes as an alternative to replacing them with spare sensors. Results show that our proposed model can result in an 18.47% MTTF increase over the triplex sensor node system with shared repair, and approximately 54.73% improvement over a non-fault tolerant (NFT) system.

Keywords: Wireless Sensor Networks, Sensor Nodes, Fault Tolerance, Mean Time to Failure, Markov Modelling

I. INTRODUCTION

Wireless Sensor Networks (WSNs) is a group of spatially distributed autonomous sensor nodes that collaborate with each other to perform an application task [1]. The proliferation of WSNs in a wide variety of application domains is in large part due to the decreasing cost and size of electronic components. Such technological advancements have made it economically feasible to measure large scale phenomena by deploying networks consisting of hundreds or thousands of nodes [2].

WSN sensor nodes are often deployed in unattended and hostile environments making them more susceptible to failures than other systems [3]. Also, manual inspection of faulty sensor nodes after deployment is typically impractical. Nevertheless, many WSN applications are mission-critical, requiring continuous operation. To meet application requirements reliably, WSNs require fault tolerance mechanisms.

Fault tolerance is the ability of a system to continue performing its intended functions in the presence of faults [4]. In a broader sense, fault tolerance is associated with reliability, with successful operation, and with the absence of failures. Thus the ultimate goal of fault tolerance is the development of a dependable system.

Understanding the likelihood of a sensor network failing at a particular point in its operational lifetime can provide valuable insight when designing the network and when considering its maintenance procedures. This paper details a Markov model for characterizing reliability and MTTF with independent repair given that the sensor node failure and subsequent repair are known. Additionally, we compare the MTTFs of our proposed model and that of the triplex sensor node Markov model with shared repair developed by Wumnaya et al. [5].

II. RELATED WORK

Fault tolerance remains an essential attribute for systems used in safety, mission and business-critical applications. In recent years, the interest in fault tolerance has been further boosted by the ongoing shift from the traditional desk-top information processing paradigm, in which a single user engages a single device for a specialized purpose, to ubiquitous computing in which many small, inexpensive networked processing devices are engaged simultaneously and distributed at all scales throughout everyday life [4][6]. As society turns into a "networked" one, it becomes increasingly important to guarantee the dependability of all services and players involved in the network.

Munir and Gordon-Ross [1] investigated the synergy of fault detection and fault tolerance for WSNs. They proposed a FT sensor node Markov model consisting of duplex sensors which contains an inactive spare sensor that can be switched on in the event of primary sensor failure. Kumar et al [7] presented a reliable and FT Markov model for a sensor network system using different types of sensors and spares that replace primary sensors in case failure occurs. Bein et al. [8] emphasized the importance of heterogeneous fault tolerance by exploring reliability issues in multi-fusion sensor networks. In [2], Markov chains are used in modelling reliability of a sensor network based on its topology given that the rates of device failure and subsequent repair are known. Wumnaya et al. [5] proposed an FT scheme for a triplex sensor node repairable system that provided a shared repair facility.

III. FAULT-TOLERANT MARKOV MODEL

In this section, we present our proposed Markov model for FT WSNs with independent repair consisting of triplex (three) sensor nodes.

III.I Fault-Tolerance Parameters

The FT parameters leveraged in the Markov model are sensor node failure rate, λ and repair rate, μ. These parameters are refered to as state transition rates since they represent the rate at which the system transits from one state to another [9]. Hence, the probabilities that cause transitions from state to state are a function of λ and μ. The behaviour of the sensor nodes are modelled and analyzed with a finite state Continuous-time Markov chain (CTMC) [4].

If $X(t)$ indicates the state of the repairable system at time t, then $\{X(t), t \geq 0\}$ is a CTMC if for all $s, t \geq 0$ and nonnegative integers i, j, k [10]:

$$P\left[X(t+s) = j \mid X(s) = i, X(u) = k, 0 \leq u \leq s\right] = P\left[X(t+s) = j \mid X(s) = i\right] \quad (1)$$

This implies that in a CTMC, the conditional probability of the future state at time $t+s$ given the present state at s and all past states depends only on the present state and is independent of the past. Let

$$p_{ij}(t) = P\left[X(t+s) = j \middle| X(s) = i\right] \quad (2)$$

and

$$p_j(t) = P\left[X(t) = j\right] \quad (3)$$

where, $p_{ij}(t)$ is the probability that the Markov chain that is presently in state i will be in state j after an additional time t, and $p_j(t)$ is the probability that a Markov chain is in state j at time t. Thus, the $p_{ij}(t)$ are the transition probability functions that satisfy the condition $0 \leq p_{ij}(t) \leq 1$

Furthermore,

$$\begin{aligned}
p_{ij}(t+s) &= \sum_{k=0}^{n} P\left[X(t+s) = j, X(t) = k \mid X(0) = i\right] \\
&= \sum_{k=0}^{n} \left\{ \frac{P\left[X(0) = i, X(t) = k, X(t+s) = j\right]}{P\left[X(0) = i\right]} \right\} \\
&= \sum_{k=0}^{n} \left\{ \frac{P\left[X(0) = i, X(t) = k\right]}{P\left[X(0) = i\right]} \right\} \left\{ \frac{P\left[X(0) = i, X(t) = k, X(t+s) = j\right]}{P\left[X(0) = i, X(t) = k\right]} \right\} \\
&= \sum_{k=0}^{n} P\left[X(t) = k \mid X(0) = i\right] P\left[X(t+s) = j \mid X(0) = i, X(t) = k\right] \\
&= \sum_{k=0}^{n} P\left[X(t) = k \mid X(0) = i\right] P\left[X(t+s) = j \mid X(t) = i, X(t) = k\right] \qquad (4) \\
&= \sum_{k=0}^{n} p_{ik}(t)\, p_{kj}(s)
\end{aligned}$$

This equation is called the Chapman-Kolmogorov equation for the CTMC and Eq. (4) is due to the Markov property. $P(t)$ can be defined as the matrix of the $p_{ij}(t)$, that is,

$$P(t)=\begin{bmatrix} p_{11}(t) & p_{12}(t) & p_{13}(t) & \dots \\ p_{21}(t) & p_{22}(t) & p_{23}(t) & \dots \\ p_{31}(t) & p_{32}(t) & p_{33}(t) & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

then the Chapman-Kolmogorov equation becomes

$$P(t+s)=P(t)P(s) \qquad (5)$$

III.II Fault-Tolerant Sensor Node Model

A three-component system has eight possible states, enumerated in Fig.1. We assume that the sensor nodes are in a parallel configuration and they can be repaired as long as the system is not in the failed or absorbing state. This means that failed sensor nodes can be repaired in states 2, 3, 4, 5, 6 and 7. An operational sensor node is one which has either never failed or has failed and been repaired (perhaps a number of times), and has not yet failed again [5].

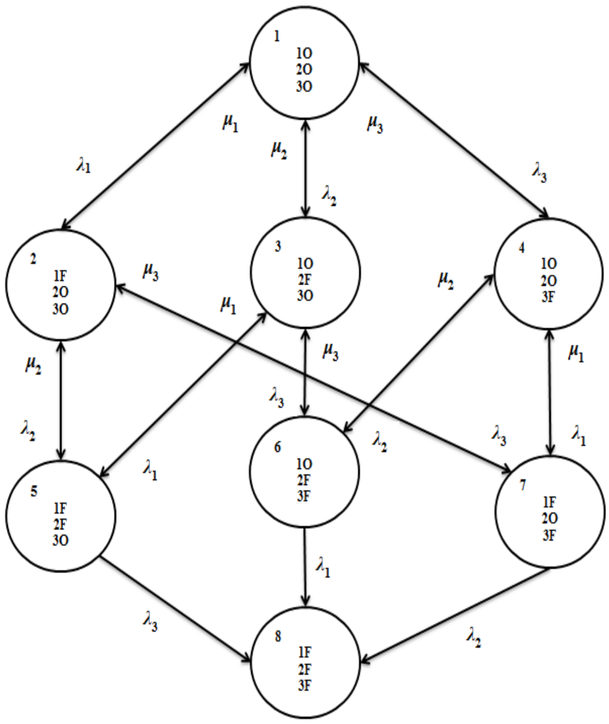


Fig.1: State Space Diagram for Triplex Sensor Node Repairable System

It is assumed that there is a fault-detection unit which detects failed or faulty sensor nodes and triggers the initiation of a repair process. Environmental and operational conditions for the system are assumed to be relatively stable as a function of time.

The failure rates λ_1, λ_2 and λ_3 of sensor nodes 1, 2 and 3 respectively, indicate the rates at which the transitions are made between states. The repair rates of sensor nodes 1, 2 and 3 are μ_1, μ_2 and μ_3 respectively. O and F indicate that a sensor node is operating or failed respectively. We follow the convention of [11] in which self-transitions are not shown. The sensor nodes are assumed to be independent of each other.

If the sensor nodes of the triplex repairable system in Fig.1 have identical failure rates $\lambda_1=\lambda_2=\lambda_3=\lambda$, and repair facilities of the same kind with repair rate μ then our FT sensor node reliability model is as shown in Fig.2. The states in the Markov model represent the number of operational sensor nodes. Since each sensor node has its own dedicated repair facility, it is assumed that there are enough repair facilities to repair more than one sensor node simultaneously, if need be, and the repair rate for any of the nodes cannot depend on how many other nodes are being repaired at the same time.

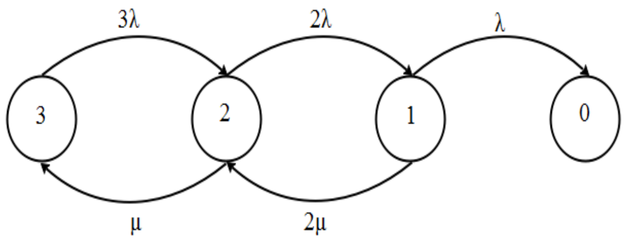


Fig.2: Triplex Sensor Node Markov Model with Independent Repair

In accordance with Fig.2, Q, the intensity matrix of the Markov model can be obtained as follows:

$$Q=\begin{bmatrix} 0 & 0 & 0 & 0 \\ \lambda & -(\lambda+2\mu) & 2\mu & 0 \\ 0 & 2\lambda & -(2\lambda+\mu) & \mu \\ 0 & 0 & 3\lambda & -3\lambda \end{bmatrix} \qquad (6)$$

The Chapman-Kolmogorov differential equations (see [10][12]) are given by:

$$\frac{dp(t)}{dt}=p(t)Q \qquad (7)$$

where

$$p(t)=\left[p_0(t), p_1(t), p_2(t), p_3(t)\right]$$

Since the matrix Q does not have full rank, we remove the first of the four equations without losing any information about $P_0(t), P_1(t), P_2(t)$ and $P_3(t)$.

The system differential equations describing the triplex sensor node Markov model employing Laplace transforms with initial conditions $P_3(0)=1,\ P_2(0)=0,\ P_1(0)=0,$ and $P_0(0)=0$ are:

$$-(\lambda+2\mu)P_1^*(s)+2\lambda P_2^*(s)=sP_1^*(s) \quad (8)$$

$$2\mu P_1^*(s)-(2\lambda+\mu)P_2^*(s)+3\lambda P_3^*(s)=sP_2^*(s) \quad (9)$$

$$\mu P_2^*(s)-3\lambda P_3^*(s)=sP_3^*(s)-1 \quad (10)$$

Solving Eqs. (8), (9) and (10) leads to $P_i^*(s)$.

The Laplace transform of the system reliability is expressed by:

$$R^*(s)=\sum_{i\in O}P_i^*(s) \quad (11)$$

where the sum is taken over all the operational states *O*.

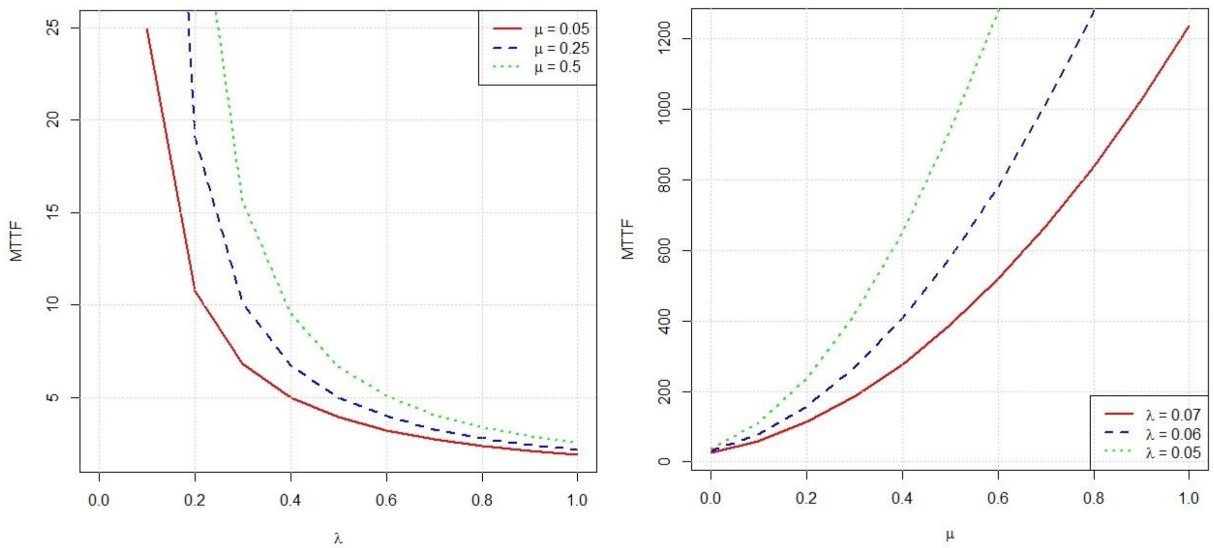
The MTTF can be achieved by evaluating $R^*(s)\big|_{s=0}$

$$MTTF=R^*(0)$$

$$=\frac{11\lambda^2+7\lambda\mu+2\mu^2}{6\lambda^3} \quad (12)$$

IV. RESULTS

We used the Maple and R Software Packages to obtain our model results. The MTTF with different values of λ and μ are shown in Fig. 3(a) and (b) respectively. As expected, in 3(a), the MTTF decreases with increasing λ for a given value of μ. However, the MTTF increases as μ increases as depicted in 3(b). With increasing μ the system becomes more reliable. When $\mu=0$, the system is non-repairable. In which case the MTTF = 26.19, 30.56 and 36.67 for $\lambda=0.07$, $\lambda=0.06$ and $\lambda=0.05$ respectively.



(a) (b)

Fig.3: Variations of MTTF with parameters (a) λ and (b) μ

Fig.4(a) depicts the MTTF for the model developed by Wumnaya et al. [5], Triplex Sensor Node System with Shared Repair ($TSNS_{Shared\_Repair}$) and our proposed model, Triplex Sensor Node Markov Model with Independent Repair ($TSNMM_{Independent\_Repair}$) when λ is set as 0.07 with varying μ. In 4(b), μ is set as 0.3 with varying λ. The comparison provides insight into how the failure rates, repair rates and number of repair facilities affect the sensor nodes' MTTF. Figure 4 shows that the MTTF for $TSNMM_{Independent\_Repair}$ is always greater than $TSNS_{Shared\_Repair}$.

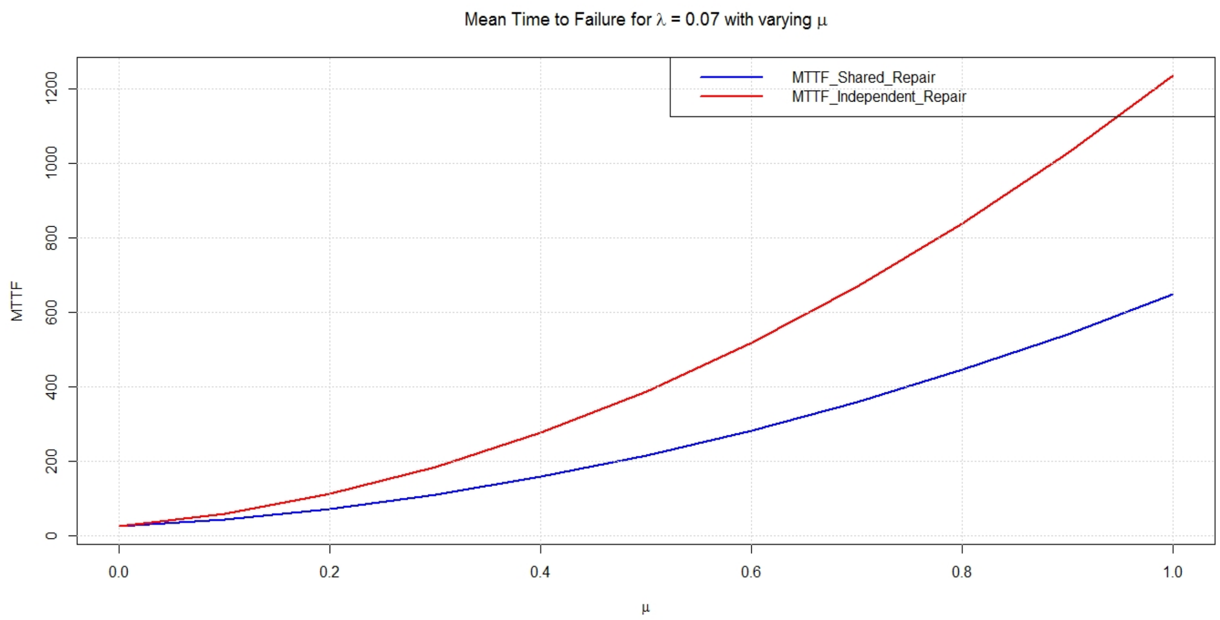


Fig.4(a): MTTF Comparison of the Two Models for $\lambda = 0.07$ with varying μ.

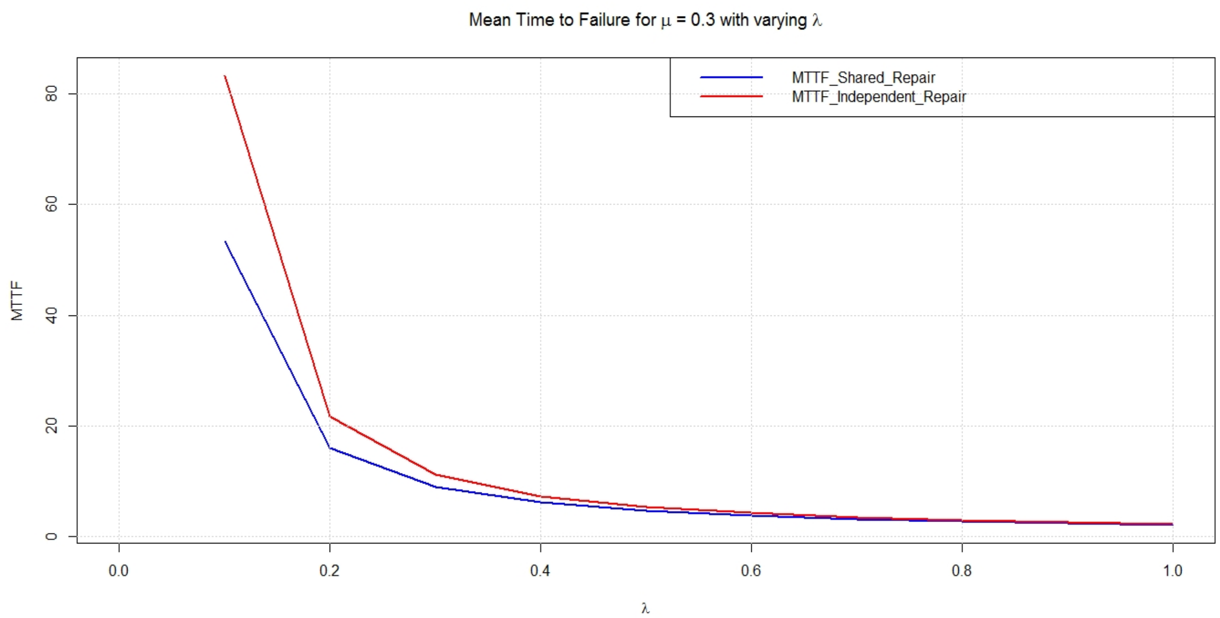


Fig.4(b): MTTF Comparison of the Two Models for $\mu = 0.3$ with varying λ.

The numerical results of the MTTFs when λ and μ are set as 0.07 and 0.05 respectively, is presented in Table 1. The percentage improvement in MTTF for $TSNMM_{Independent\_Repair}$ over the $TSNS_{Shared\_Repair}$ is 18.47%, and approximately 54.73% improvement over a non-fault tolerant (NFT) system.

Table 1: Comparison of MTTF for the Two Models

Model	**FT**	**NFT**
$TSNS_{Shared\_Repair}$	34.208	26.190
$TSNMM_{Independent\_Repair}$	40.525	26.190

V. CONCLUSION

In this paper, we proposed a FT triplex sensor node Markov model with independent repair facilities. We observed that the MTTF is sensitive to changes in λ and μ and that the effect of the repair facility increased the mean life by $\frac{7\lambda\mu + 2\mu^2}{6\lambda^3}$.

Comparison results indicate that our proposed FT triplex sensor node Markov model with independent repair provides 18.47% MTTF improvement over the triplex sensor node system with shared repair developed by Wumnaya et al. [5], and approximately 54.73% improvement over a NFT system.

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