

# Multi-Objective Optimization of User Capacity for WPCN

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## Abstract

In this paper our goal is to maximize the individual user capacities of massive multi-input multi-output (MU-MIMO) systems through wireless powered communication network (WPCN). A multiple-objective optimization approach (MOOP) is developed while guaranteeing the quality of service (QoS) requirement. A new concept is proposed to derive Pareto solution via weighted Tchebycheff method from an equivalent single-objective optimization problem (SOOP) that minimizes the maximum of several quasiconvex fractional functions by iterative algorithm. The simulation results show that the proposed algorithm has the optimal performance and the user capacity of the distributed massive (DM) multi-input multi-output (MIMO) is considerably higher than that of the centralized massive (CM)-MIMO system.

**Keywords** - Multiobjective optimization, Pareto optimal, quasiconvex, wireless powered communication networks

## I. INTRODUCTION

Wireless powered communication network (WPCN) is a novel paradigm in which the battery of wireless communication devices can be remotely replenished by means of microwave wireless power transfer (WPT) technology. By WPCN, replacement of manual battery is reduced, and thus considerably improves the performance over conventional communication networks in many aspects, such as better throughput, higher device lifetime, and lower network operating cost [1].

In WPCN, wireless terminals are powered by the radio signals for information transmission. The terminals harvest energy from radiated signals by transmitters and then transmit information signals by using the harvested energy [2]. Though, the transmission efficiency of the WPT reduces rapidly depending on the transmission distance. Conveniently, the distributed massive (DM) multi-input multi-output (MIMO) system has a capability of solving the double near-far problem [3, 4], where a user which is far from the radio remote heads (RRHs) has receives less power than a user which is near to RRH in the downlink (DL) energy transfer. In the uplink (UL) information transmission more signal power is attenuated due to path loss. Such a path loss can be reduced because RRH are geographically distributed in the DM-MIMO. Also, DM-MIMO system can achieve high frequency efficiency [5, 6] and energy efficiency [6].

For comparison, we also study the case of multi-user centralized massive (CM) MIMO system where the base station operates in half duplex (HD) mode. Remarkably, unlike CM-MIMO, the DM-MIMO system consists of geographically distributed radio remote heads (RRHs) that

mitigates the inherent channel propagation loss, and thus can achieve significantly higher energy and frequency efficiencies.

In this paper, we study in WPCN to improve and optimize the user capacity. The capacity of every single user should be separately measured. The capacities of these different users cannot be optimized simultaneously. The influences of this paper are shortened as follows.

- Multi-objective optimization problem (MOOP) is formulated to set of Pareto optimal solution [8]. We adopt a weighted Tchebycheff method to find Pareto optimal of each user capacity to convert MOOP into a single objective optimization problem (SOOP).
- The SOOP is a generalized fractional programming (GFP) problem [9, 10]. Iterative algorithm is used to solve above method. Simulation results are provided to compare the user capacity performance and confirm that this method succeeds in achieving better outcome with fast convergence speed.

The rest of this paper is organized as follows. The system model and the problem formulation is presented in Section II. In Section III, the problem will be studied and by iterative algorithm we get its solution. Sections IV shows the simulation results and we finally conclude in Section V.

## II. SYSTEM MODEL

We consider a WPCN model containing of  $N$  RRHs equipped with  $M$  antennas and  $K$  users with a single antenna. We assumed radius of hexagon cell is 1m. The RRHs are connected to a baseband process unit (BPU) via optical fibers. In WPCN model, both wireless energy transfer (WET) in DL mode and wireless information transmission (WIT) in the UL mode are coordinated by a central processing unit. A frame-based transmission is done and without loss of generality, it is normalized to 1. It is supposed that the channel matrix is known perfectly at the BPU. In each block, in the first duration  $\theta$  ( $0 < \theta < 1$ ) time, the RRHs in the DL mode broadcast energy signals to transfer energy to all users simultaneously, while in the remaining  $(1 - \theta)$  amount of time of the block, all users transfer their independent information to the RRHs simultaneously in the UL using their harvested energy.

The channel vector from all the RRHs to the  $k$ th user is expressed by

$$\mathbf{g}_k = \Lambda_k^{1/2} \mathbf{h}_k \quad (1)$$

where  $\Lambda_k = \text{diag}([\zeta_{1,k}, \dots, \zeta_{N,k}]) \otimes \mathbf{I}_M$  and

$$\mathbf{h}_k = [\mathbf{h}_{1,k}^T, \dots, \mathbf{h}_{N,k}^T]^T.$$

Here  $(\cdot)^T$  is the transpose and  $\zeta_{n,k}$  is the path loss of the channel between the  $n$ th RRH and the  $k$ th user.  $\otimes$  is the Kronecker product and  $\mathbf{h}_{n,k}$  is the  $M \times 1$  independent Rayleigh fading coefficients between the  $n$ th RRH and the  $k$ th user. In the DL phase, assuming channel reciprocity, the received signal  $x_k$  at the  $k$ th user can be given by

$$x_k = \sqrt{p_k} \mathbf{g}_k^H \mathbf{w} + n_k \quad (2)$$

where  $p_k$  is the DL transmission power to the  $k$ th user,  $\mathbf{w} = \sum_{k=1}^K \mathbf{u}_k$ , and  $n_k$  is zero-mean additive white Gaussian noise with variance  $\sigma_d^2$ . Here  $\mathbf{u}_k$  is the DL energy beamforming vector for the  $k$ th user. The noise power is too small for energy harvesting compared with the received signal power. Therefore, the harvested energy at the  $k$ th user is written as

$$E_k = \varepsilon \theta \mathbb{E}[|x_k|^2] = \varepsilon \theta p_k \mathbb{E}[|\mathbf{g}_k^H \mathbf{w}|^2] \quad (3)$$

where  $\mathbb{E}[\cdot]$  denotes the statistical expectation and  $0 < \varepsilon \leq 1$  is the energy conversion efficiency. The received signal vector for UL phase is given by

$$\mathbf{r} = \mathbf{G}\mathbf{s} + \mathbf{z} \quad (4)$$

Here  $\mathbf{G} = [\mathbf{g}_1, \dots, \mathbf{g}_K]$ ,  $\mathbf{s}$  is the information carrying signals of the users, and  $\mathbf{z}$  is the receiver noise vector with zero mean and variance  $\sigma_u^2$ . The BPU decodes the received signals from the  $k$ th user via a receive beamforming vector denoted by  $\mathbf{v}_k$ ,  $k = 1, \dots, K$ . Thus, the achievable UL capacity for the  $k$ th user is given by

$$C_k = (1 - \theta) \log_2(1 + \gamma_k) \quad (5)$$

where  $\gamma_k$  is the signal to interference plus noise ratio (SINR) given by

$$\gamma_k \triangleq \frac{S_k}{I_k} = \frac{q_k |\mathbf{v}_k^H \mathbf{g}_k|^2}{\sum_{i=1, i \neq k}^K q_i |\mathbf{v}_k^H \mathbf{g}_i| + |\mathbf{v}_k^H \mathbf{z}| \sigma_u^2} \quad (6)$$

Here  $q_k$  denotes the average UL transmit power for the  $k$ th user

In this section, we are interested in maximizing the capacity of all individual users by optimizing over time allocation ( $\theta$ ), transmit power ( $\mathbf{p}$ ) in UL and beamforming vectors ( $\mathbf{w}$ ,  $\mathbf{V}$ ), i.e.,

$$\max_{\theta, \mathbf{p}, \mathbf{w}, \mathbf{V}} C_k \quad (7)$$

$$C1: 0 < \theta < 1 \quad (8)$$

$$C2: (1 - \theta) q_k = E_k, \quad \forall k \quad (9)$$

$$C3: \sum p_k < p_{max} \quad (10)$$

$$C4: \|\mathbf{w}\| = 1 \quad (11)$$

where  $\mathbf{p} = [p_1, \dots, p_K]$  and  $\mathbf{V} = [\mathbf{v}_1, \dots, \mathbf{v}_K]$ .

### III. THE PROPOSED ALGORITHM

In this section, an algorithm is proposed to find the Pareto optimal of user capacity by converting the MOOP in (7) into a SOOP. This MOOP function is non-convex due to the coupled variables and UL transmit power constraints, which can be

converted into single objective optimization [7, 8] using the weighted Tchebycheff method expressed as

$$\max_{\theta, \mathbf{w}, \mathbf{p}, \mathbf{V}} \min_k \left\{ \varphi_k \left( C_k^0 - C_k \right) \right\} \quad (12)$$

subject to (8)-(11). Here,  $\boldsymbol{\varphi} = \{\varphi_1, \dots, \varphi_K\}$  is positive weighting vector and  $C_k^0$  is the Utopia capacity of user  $k$ . Further, the above objective function (12) is quasiconvex and can be solved by separating  $\mathbf{w}$  from others since DL beamformer only affects amount of energy harvesting as per (3). Let  $\mathbf{u}_k^*$  represents the optimal beamforming vector for maximizing the harvested energy of user  $k$  which is the dominant eigenvector of  $\mathbf{g}_k \mathbf{g}_k^H$ . Thus, the proposed optimal downlink beamforming vector is given by

$$\mathbf{w}^* = \sum_{k=1}^K \frac{1}{K} \frac{\mathbf{u}_k^*}{\|\mathbf{u}_k^*\|} \quad (13)$$

Next, fixing  $\theta = \bar{\theta}$  and substituting for  $\gamma_k$  in objective function (12) is equivalent to the following:

$$\min_{\mathbf{p}, \mathbf{V}} \max_k \left\{ \varphi_k \left( \frac{\gamma_k^0 I_k - S_k}{I_k} \right) \right\} \quad (14)$$

The above expression (14) is a generalized fractional programming (GFP), which minimizes the maximum of numerous fractions [8, 10]. Using the following method, equation (14) can be transformed to better tractable one, i.e., the objective function (14) is quasiconvex and equivalent to [7],

$$\max_{\mathbf{y} \in \mathbf{Y}} \min_{\mathbf{p}, \mathbf{V}} f(\mathbf{y}, \mathbf{p}, \mathbf{V}) = \frac{\sum_{k=1}^K y_k \varphi_k (\gamma_k^0 I_k - S_k)}{\sum_{k=1}^K y_k I_k} \quad (15)$$

where  $\mathbf{Y} \triangleq \left\{ (y_1, \dots, y_K) \mid y_k \geq 0, \forall k, \sum_{k=1}^K y_k = 1 \right\}$ .

The optimization problem (15) can be solved by iteratively finding the solutions of the following two subproblems: finding the optimal  $\{\mathbf{p}^*, \mathbf{V}^*\}$  for a given  $\mathbf{y}$ , and finding the optimal  $\mathbf{y}$ . Let  $\gamma(\mathbf{y}) = \min_{\mathbf{p}, \mathbf{V}} f(\mathbf{y}, \mathbf{p}, \mathbf{V})$  and  $\gamma^* = \max_{\mathbf{y} \in \mathbf{Y}} \gamma(\mathbf{y})$ .

We define a function as

$$U(\mathbf{y}, \alpha) = \sum_{k=1}^K y_k \left( \varphi_k \left( (\gamma_k^0 I_k - S_k) - \alpha I_k \right) \right) \quad (16)$$

Let  $\mathbf{y}^{(n)}$ ,  $n = 0, 1, \dots$  be a sequence updated by the following equation for any initial  $\mathbf{y}^{(0)}$

$$\mathbf{y}^{(n+1)} = \arg \max_{\mathbf{y} \in \mathbf{Y}} \min_{\mathbf{p}, \mathbf{V}} U(\mathbf{y}, \gamma(\mathbf{y}^{(n)})) \quad (17)$$

The optimal solutions can be achieved as follows:

- (a) when  $\min_{\mathbf{p}, \mathbf{V}} U(\mathbf{y}, \gamma(\mathbf{y})) = 0$ ,  $\gamma(\mathbf{y})$  is obtained.
- (b) when  $\gamma(\mathbf{y}^{(n+1)}) = \gamma(\mathbf{y}^{(n)})$ ,  $\gamma^* = \gamma(\mathbf{y}^{(n)})$  is obtained.

For the above functions Dinkelbach algorithm is used to find the optimal solution [9].

We can solve the problem (14) by the iterative algorithm in Table I. In the table, the first subproblem are solved in steps 2 to 9 whereas steps 10 to 17 gives the solution of the second subproblem.  $\varepsilon_1$  and  $\varepsilon_2$  are small values, respectively.

**Table 1** Algorithm to find Pareto Optimal

1. Initialize $\mathbf{y}^{(0)} \in Y, \gamma, n = 0, \varepsilon_1$ and $\varepsilon_2$
2. Find $\{\mathbf{p}^*, \mathbf{V}^*\} = \arg \min_{\mathbf{p}, \mathbf{V}} U(\mathbf{y}^{(n)}, \gamma)$ .
3. <b>If</b> $ \sum_{k=1}^K y_k^{(n)} \varphi_k(\gamma_k^0 I_k^* - S_k^*) - \gamma I_k^*  < \varepsilon_1$ , <b>then</b>
4.     set $\gamma(\mathbf{y}^n) = \gamma$ .
5. <b>goto</b> step (10).
6. <b>else</b>
7.     update $\gamma = \frac{\sum_{k=1}^K y_k^{(n)} \varphi_k(\gamma_k^0 I_k^* - S_k^*)}{\sum_{k=1}^K y_k^{(n)} I_k^*}$ .
8. <b>goto</b> step (2).
9. <b>end</b>
10. Update $\mathbf{y}^{(n+1)} = \arg \max_{\mathbf{y} \in Y} \min_{\mathbf{p}, \mathbf{V}} U(\mathbf{y}, \gamma(\mathbf{y}^{(n)}))$ .
11. <b>If</b> $\gamma(\mathbf{y}^{(n+1)}) - \gamma(\mathbf{y}^{(n)}) < \varepsilon_2$ , <b>then</b>
12.     set $\gamma^* = \gamma(\mathbf{y}^{(n)})$ .
13. <b>exit</b> .
14. <b>else</b>
15.     update $n = n + 1$ .
16. <b>goto</b> step (2).
17. <b>End</b>

The first subproblem for given  $\mathbf{y}$  in step 2 can be written as

$$\max_{\mathbf{p}, \mathbf{V}} \sum_{k=1}^K y_k \left( \varphi_k \left( (\gamma_k^0 I_k - S_k) - \alpha I_k \right) \right) \quad (18)$$

*s.t.* C1 – C4

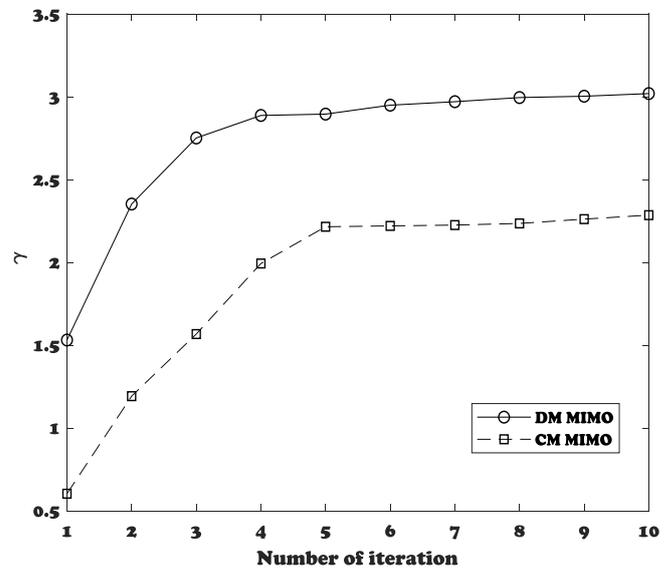
The second subproblem in step 10 can be written as

$$\max_{\mathbf{y} \in Y} \min_{\mathbf{p}, \mathbf{V}} \sum_{k=1}^K y_k \left( \varphi_k (\gamma_k^0 I_k - S_k) - \alpha I_k \right) \quad (19)$$

The above optimization problem is convex. Accordingly, a classic convex optimization method is used to solve it [11].

#### IV. SIMULATION RESULT

We consider a hexagonal cell which has randomly distributed user  $K$ . Seven RRHs are distributed with radius  $r_1 = 0, r_2 = \dots = r_7 = (3 - \sqrt{3})/2$  and angles  $\theta_1 = 0, \theta_2 = \pi/6, \theta_3 = 3\pi/6, \theta_4 = 5\pi/6, \theta_5 = 7\pi/6, \theta_6 = 9\pi/6, \theta_7 = 11\pi/6$ .  $\sigma_u^2 = \sigma_d^2 = -50dBm$  with path loss model  $\xi_{n,k} = 10^{-3} d_{n,k}^{-3}$  where  $d_{n,k}$  is the distance between the user  $k$  and the RRH  $n$ . RRH is equipped with  $M = 50$  antennas and the total number of antennas is  $MN = 350$ , which is assumed to be same both in CM-MIMO and DM-MIMO. We assumed  $K = 2$  users that are uniformly distributed.



**Fig. 1** Convergence of SINR in DM- MIMO and CM-MIMO

Fig. 1 shows the convergence of  $\gamma(\mathbf{y}^n)$ . One can see that the proposed algorithm has a faster convergence speed. Also the DM-MIMO shows better performance compared with the CM-MIMO.

Fig. 2 demonstrates the user's capacity versus the total maximum transmission power in both CM-MIMO and DM-MIMO scenario. It is noticeable that the DM-MIMO achieves higher user capacity compared with the CM-MIMO. It is seen that both DM-MIMO and CM-MIMO increases the user capacity according to the total maximum transmission power.

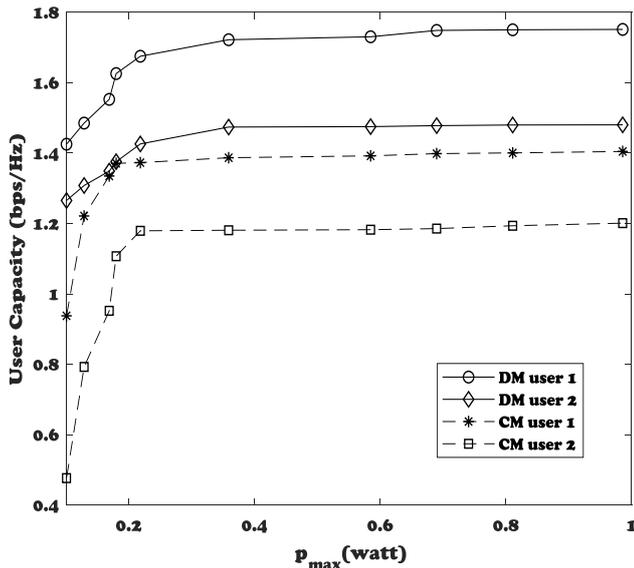


Fig.2. Comparison of user capacity versus  $p_{\max}$  for DM-MIMO and CM-MIMO.

## V. CONCLUSIONS

In this paper, we considered the multiobjective optimization for user capacity in WPCN network. To find Pareto optimal, the multiobjective optimization problem is converted into a single objective problem using Tchebycheff method. Then it is solved by iterative algorithm. It is shown that the proposed algorithm achieves a higher user capacity for both CM-MIMO and DM-MIMO. Further, the user capacity of the DM-MIMO is significantly higher compared with the CM-MIMO.

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## REFERENCES

- [1] S. Bi, C. K. Ho, & R. Zhang, (2015). Wireless powered communication: opportunities and challenges. *IEEE Commun. Mag.*, **53**(4), 117-125
- [2] Zhang, L., Liang, Y. C., & Xin, Y.,(2008). Joint beamforming and power control for multiple access channels in cognitive radio networks. *IEEE J. Sel. Areas Commun.*, **26** (1), 38-51.
- [3] Yuan, F. (2015). Joint wireless information and energy transfer in massive distributed antenna systems. *IEEE Commun. Mag.*, **53** (6), 109-116.
- [4] Ju, H., and Zhang, R., (2014). Throughput maximization in wireless powered communication networks. *IEEE Trans. Wirel. Commun.*, **13** (1), 418-428.
- [5] Lee, S. R., Moon, J. S., Kim S. H. and I. Lee, (2012). Capacity analysis of distributed antenna systems in a composite fading channel. *IEEE Trans. Wirel. Commun.*, **11** (3), 1076-1086
- [6] Kim, W. and Yoon, W. (2017). Joint Beam Forming and Resource Allocation for WPCN with Distributed Massive MIMO System. *IJEAR*, **12** (20), 10073-10076
- [7] Yu, G., Jiang, Y., Xu, L., and Li, G. Y.(2015). *Multi-objective energy-efficient resources allocation for multi-RAT heterogeneous networks*. *IEEE J. Sel. Areas Commun.*, **13** (10), pp. 2118-2126
- [8] Marler, R.T., and Arora, J.S. (2004). Survey of multi-objective optimization methods for engineering. *Struct. Multidisc. Opti.*, **6** (6), pp. 369-395
- [9] Dinkelbach, W. (1967), On nonlinear fractional programming. *Manage. Sci.*, **13**(7), 492-498
- [10] Crouzeix, J.P, and Ferland, J.A. (1991). Algorithms for generalized fractional programming', *Math. Program* , **52** (2), 191-207
- [11] Barros, A. I., Frenk, J. B. G., Schaible, S. and Zhang, S. (1996). A new algorithm for generalized fractional programs. *Math. Program*, **72** (2), 147-175.