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Deteriorating Items Supply Chain Inventory Model for Single Vendor Single Buyer under Time and Price Dependent Demand

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Abstract

An optimal policy for vendor and buyer is developed for price and time dependent demand function and items having deterioration. Single vendor single buyer system model is obtained as profit maximization for determining optimal cycle time (strategy) of system. We also determine the profit of buyer-vendor jointly. Numerical illustrations show that both buyer and vendor earn significant profit in supply chain inventory system. Post-optimality analysis for parameters is also carried out.

Keywords

Supply chain, Optimal strategy, Deterioration, Time dependent demand, Price dependent demand

I. INTRODUCTION

In past researchers have focused for the improvement of joint buyer vendor inventory framework with various assumptions on demand pattern, for example, price dependent, time dependent demand and so on for minimizing vendor buyer total relevant costs. Supply chain management is concerned with coordination between manufacturer, wholesalers and the retailers to maximize the system's profit. For maximizing supplier's economic gain with no added cost to the buyer, Monahan (1984) developed a vendor oriented optimal quantity discount policy. A joint profit sharing plan between the vendor and the buyer was considered by Chakravarty and Martin (1988). Under a periodic review for any desirable negotiation factor the algorithm determines both discount price and replenishment interval. A joint decision policy in which unit selling price and order quantity is coordinated through quantity discount and franchise fees was considered by Weng (1995). Chakravarty and Martin's (1988) model was extended by Wee (1998) by introducing deterioration and obtained optimal buyer seller discount pricing under price dependent demand. Yang and Wee (2003) developed a profit sharing integrated model when demand is price sensitive. An integrated supply chain model with one vendor and multiple buyers was developed by Yau and Chiao (2004) to minimize vendor's total annual cost subject to maximum cost customers are ready to pay. An integrated production inventory model for deteriorating items for one vendor one buyer was developed by Zhou and Wang (2007). One vendor multi-buyers EOQ integrated inventory system having multiple shipping strategies was considered by Shaw et al. (2012). By considering lead time following normal distribution and demand is price dependent, a supply chain inventory model with one producer and many buyers has been developed by Giri and Roy (2015). Ghiami and Williams (2015) delivered a deteriorating item models with multiple buyers and single manufacturer in a supply chain production inventory system with finite production rate. For use of activity based costing approach in supply chain management and cost managing for ordering inventory was given by Momeni and Azizi (2018).

A one vendor one buyer combined inventory models for varying deterioration for buyer and time varying holding cost for vendor buyer both under time and price dependent demand is considered in this paper. We assume that vendor has better preservation technology, so preservation technology cost is included for vendor and therefore there is no deterioration cost for vendor.

II. NOTATIONS AND ASSUMPTIONS

Notations:

For obtaining model, list of notations used are:

$D(t)$: $a + bt - \rho p$, where $a > 0$, $0 < b < 1$, $p > 0$, $\rho > 0$

$I_b(t)$: Buyer's inventory size at any time t

$I_v(t)$: Inventory size of vendor at any t

A_b : Per order ordering cost of buyer

A_v : Per order ordering cost of vendor

c_b : Per unit cost of purchasing of buyer

θ : Deterioration rate during $t_1 \le t \le t_2$, $0< \theta<1$

θt : Deterioration rate during , $t_2 \le t \le T$, $0< \theta<1$

x_b : Fixed holding cost of buyer

y_b : Varying holding cost of buyer

x_v : Fixed holding cost of vendor

y_v : Varying holding cost of vendor

p : Per unit selling price of buyer (a decision variable)

m : Preservation technology cost for vendor (fixed)

n : Number of time orders placed by buyer during cycle time.

TP_b : Total profit of buyer

TP_v : Total profit of vendor

TP : Vendor buyer's total profit

$t_1=v_1*T_b$, $t_2=v_2*T_b$, where $T_b = T/n$

T : Cycle time of vendor (a decision variable).

Assumptions:

For developing model, assumptions considered are:

- Demand is function of time and price.
- One vendor and one buyer are considered.
- Stock out is not permitted.
- Lead time is zero.
- During the cycle time, no repairing or replacement of deteriorated units and deterioration is dependent on time for buyer's inventory.

- For buyer and vendor both, time varying holding cost is considered.

III. THE MATHEMATICAL MODEL AND ANALYSIS

Let inventory level at time t be given by $I_b(t)$ $(0 \le t \le T_b)$ as shown below.

Buyer's Inventory

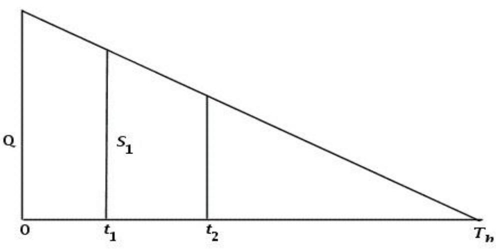

Figure 1

Two situations are discussed. The situation one there is no collaboration between vendor and buyer, while in situation two there is collaboration of buyer and vendor. Considering linear demand, inventory size is given for buyer and vendor. Change in inventory sizes are given by following differential equations for vendor and buyer:

$$\frac{dI_b(t)}{dt} = -(a + bt - \rho p), \qquad 0 \le t \le t_1 \qquad (1)$$

$$\frac{dI_b(t)}{dt} + \theta I_b(t) = -(a + bt - \rho p), \qquad t_1 \le t \le t_2 \qquad (2)$$

$$\frac{dI_b(t)}{dt} + \theta t I_b(t) = -(a + bt - \rho p), \qquad t_2 \le t \le T_b \qquad (3)$$

$$\frac{dI_v(t)}{dt} = -(a + bt - \rho p), \qquad 0 \le t \le T \qquad (4)$$

with initial conditions $I_b(0) = Q$, $I_b(t_1) = S_1$, $I_b(T_b) = 0$ and $I_v(T) = 0$.

These equations have solutions:

$$I_b(t) = Q - (at - \rho pt + \frac{1}{2}bt^2), \qquad (5)$$

$$I_b(t) = \begin{bmatrix} a(t_1 - t) - \rho p(t_1 - t) + \frac{1}{2}a\theta(t_1^2 - t^2) \\ - \frac{1}{2}\rho p\theta(t_1^2 - t^2) + \frac{1}{2}b(t_1^2 - t^2) \\ + \frac{1}{3}b\theta(t_1^3 - t^3) - a\theta t(t_1 - t) \\ + \rho p\theta t(t_1 - t) - \frac{1}{2}b\theta t(t_1^2 - t^2) \end{bmatrix} + S_1\left[1 + \theta(t_1 - t)\right] \qquad (6)$$

$$I_b(t) = \begin{bmatrix} a(T_b - t) - \rho p(T_b - t) + \frac{1}{2}b(T_b^2 - t^2) \\ + \frac{1}{6}a\theta(T_b^3 - t^3) - \frac{1}{6}\rho p\theta(T_b^3 - t^3) \\ + \frac{1}{8}b\theta(T_b^4 - t^4) - \frac{1}{2}a\theta t^2(T_b - t) \\ + \frac{1}{2}\rho p\theta t^2(T_b - t) - \frac{1}{4}b\theta t^2(T_b^2 - t^2) \end{bmatrix}. \qquad (7)$$

$$I_v(t) = \left[a(T - t) - \rho p(T - t) + \frac{1}{2}b(T^2 - t^2)\right]. \qquad (8)$$

(not considering higher powers of θ)

Substituting $t = t_1$ in equation (5), we get

$$Q = S_1 + \left(at_1 - \rho pt_1 + \frac{1}{2}bt_1^2\right). \qquad (9)$$

Putting $t = t_2$ in equations (6) and (7), we have

$$I_b(t_2) = \begin{bmatrix} a(t_1 - t_2) - \rho p(t_1 - t_2) + \frac{1}{2}a\theta(t_1^2 - t_2^2) \\ - \frac{1}{2}\rho p\theta(t_1^2 - t_2^2) + \frac{1}{2}b(t_1^2 - t_2^2) \\ + \frac{1}{3}b\theta(t_1^3 - t_2^3) - a\theta t_2(t_1 - t_2) \\ + \rho p\theta t_2(t_1 - t_2) - \frac{1}{2}b\theta t_2(t_1^2 - t_2^2) \end{bmatrix} + S_1\left[1 + \theta(t_1 - t_2)\right] \qquad (10)$$

$$I_b(t_2) = \begin{bmatrix} a(T_b - t_2) - \rho p(T_b - t_2) + \frac{1}{2}b(T_b^2 - t_2^2) \\ + \frac{1}{6}a\theta(T_b^3 - t_2^3) - \frac{1}{6}\rho p\theta(T_b^3 - t_2^3) \\ + \frac{1}{8}b\theta(T_b^4 - t_2^4) - \frac{1}{2}a\theta t^2(T_b - t_2) \\ + \frac{1}{2}\rho p\theta t_2^2(T_b - t_2) - \frac{1}{4}b\theta t_2^2(T_b^2 - t_2^2) \end{bmatrix}. \qquad (11)$$

So from equations (10) and (11), we get

$$S_1 = \frac{1}{\left[1+\theta\left(t_1 - t_2\right)\right]}\left[\begin{array}{l} a\left(T_b - t_2\right) - \rho p\left(T_b - t_2\right) \\ +\frac{1}{2}b\left(T_b^2 - t_2^2\right)+\frac{1}{6}a\theta\left(T_b^3 - t_2^3\right) \\ -\frac{1}{6}\rho p\theta\left(T_b^3 - t_2^3\right)+\frac{1}{8}b\theta\left(T_b^4 - t_2^4\right) \\ -\frac{1}{2}a\theta t^2\left(T_b - t_2\right)+\frac{1}{2}\rho p\theta t_2^2\left(T_b - t_2\right) \\ -\frac{1}{4}b\theta t_2^2\left(T_b^2 - t_2^2\right)- a\left(t_1 - t_2\right) \\ +\rho p\left(t_1 - t_2\right)-\frac{1}{2}a\theta\left(t_1^2 - t_2^2\right) \\ +\frac{1}{2}\rho p\theta\left(t_1^2 - t_2^2\right)-\frac{1}{2}b\left(t_1^2 - t_2^2\right) \\ -\frac{1}{3}b\theta\left(t_1^3 - t_2^3\right)+ a\theta t_2\left(t_1 - t_2\right) \\ -\rho p\theta t_2\left(t_1 - t_2\right)+\frac{1}{2}b\theta t_2\left(t_1^2 - t_2^2\right) \end{array}\right]. \tag{12}$$

Using equation (12) into equation (6), we have

$$I_b(t) = \left[\begin{array}{l} a\left(t_1 - t\right)-\rho p\left(t_1 - t\right)+\frac{1}{2}a\theta\left(t_1^2 - t^2\right) \\ -\frac{1}{2}\rho p\theta\left(t_1^2 - t^2\right)+\frac{1}{2}b\left(t_1^2 - t^2\right) \\ +\frac{1}{3}b\theta\left(t_1^3 - t^3\right)- a\theta t\left(t_1 - t\right) \\ +\rho p\theta t\left(t_1 - t\right) - \frac{1}{2}b\theta t\left(t_1^2 - t^2\right) \end{array}\right]$$

$$+\frac{\left[1+\theta\left(t_1 - t\right)\right]}{\left[1+\theta\left(t_1 - t_2\right)\right]}\left[\begin{array}{l} a\left(T_b - t_2\right) - \rho p\left(T_b - t_2\right) \\ +\frac{1}{2}b\left(T_b^2 - t_2^2\right)+\frac{1}{6}a\theta\left(T_b^3 - t_2^3\right) \\ -\frac{1}{6}\rho p\theta\left(T_b^3 - t_2^3\right)+\frac{1}{8}b\theta\left(T_b^4 - t_2^4\right) \\ -\frac{1}{2}a\theta t^2\left(T_b - t_2\right)+\frac{1}{2}\rho p\theta t_2^2\left(T_b - t_2\right) \\ -\frac{1}{4}b\theta t_2^2\left(T_b^2 - t_2^2\right)- a\left(t_1 - t_2\right) \\ +\rho p\left(t_1 - t_2\right)-\frac{1}{2}a\theta\left(t_1^2 - t_2^2\right) \\ +\frac{1}{2}\rho p\theta\left(t_1^2 - t_2^2\right)-\frac{1}{2}b\left(t_1^2 - t_2^2\right) \\ -\frac{1}{3}b\theta\left(t_1^3 - t_2^3\right)+ a\theta t_2\left(t_1 - t_2\right) \\ -\rho p\theta t_2\left(t_1 - t_2\right)+\frac{1}{2}b\theta t_2\left(t_1^2 - t_2^2\right) \end{array}\right] \tag{13}$$

Putting value of S_1 from (12) in (9), we have

$$Q = \frac{1}{\left[1+\theta\left(t_1 - t_2\right)\right]}\left[\begin{array}{l} a\left(T_b - t_2\right) - \rho p\left(T_b - t_2\right) \\ +\frac{1}{2}b\left(T_b^2 - t_2^2\right)+\frac{1}{6}a\theta\left(T_b^3 - t_2^3\right) \\ -\frac{1}{6}\rho p\theta\left(T_b^3 - t_2^3\right)+\frac{1}{8}b\theta\left(T_b^4 - t_2^4\right) \\ -\frac{1}{2}a\theta t^2\left(T_b - t_2\right)+\frac{1}{2}\rho p\theta t_2^2\left(T_b - t_2\right) \\ -\frac{1}{4}b\theta t_2^2\left(T_b^2 - t_2^2\right)- a\left(t_1 - t_2\right) \\ +\rho p\left(t_1 - t_2\right)-\frac{1}{2}a\theta\left(t_1^2 - t_2^2\right) \\ +\frac{1}{2}\rho p\theta\left(t_1^2 - t_2^2\right)-\frac{1}{2}b\left(t_1^2 - t_2^2\right) \\ -\frac{1}{3}b\theta\left(t_1^3 - t_2^3\right)+ a\theta t_2\left(t_1 - t_2\right) \\ -\rho p\theta t_2\left(t_1 - t_2\right)+\frac{1}{2}b\theta t_2\left(t_1^2 - t_2^2\right) \end{array}\right] + \left(at_1 - \rho p t_1 + \frac{1}{2}bt_1^2\right) \tag{14}$$

Putting value of Q in (5), we have

$$I_b(t) = \frac{1}{\left[1+\theta\left(t_1 - t_2\right)\right]}\left[\begin{array}{l} a\left(T_b - t_2\right) - \rho p\left(T_b - t_2\right) \\ +\frac{1}{2}b\left(T_b^2 - t_2^2\right)+\frac{1}{6}a\theta\left(T_b^3 - t_2^3\right) \\ -\frac{1}{6}\rho p\theta\left(T_b^3 - t_2^3\right)+\frac{1}{8}b\theta\left(T_b^4 - t_2^4\right) \\ -\frac{1}{2}a\theta t^2\left(T_b - t_2\right)+\frac{1}{2}\rho p\theta t_2^2\left(T_b - t_2\right) \\ -\frac{1}{4}b\theta t_2^2\left(T_b^2 - t_2^2\right)- a\left(t_1 - t_2\right) \\ +\rho p\left(t_1 - t_2\right)-\frac{1}{2}a\theta\left(t_1^2 - t_2^2\right) \\ +\frac{1}{2}\rho p\theta\left(t_1^2 - t_2^2\right)-\frac{1}{2}b\left(t_1^2 - t_2^2\right) \\ -\frac{1}{3}b\theta\left(t_1^3 - t_2^3\right)+ a\theta t_2\left(t_1 - t_2\right) \\ -\rho p\theta t_2\left(t_1 - t_2\right)+\frac{1}{2}b\theta t_2\left(t_1^2 - t_2^2\right) \end{array}\right] + a\left(t_1 - t\right) - \rho p\left(t_1 - t\right) + \frac{1}{2}b\left(t_1^2 - t^2\right). \tag{15}$$

Total relevant profit consisted of following elements:

Buyer's relevant costs:

(i) Ordering cost (OC_b) = n A_b (16)

(ii) Holding Cost:

$$HC_b = n\int_0^{T_b} (x_b + y_b t) I_b(t)dt$$

$$= n\left[\int_0^{t_1} (x_b + y_b t)\, I_b(t)dt + \int_{t_1}^{t_2} (x_b + y_b t)\, I_b(t)dt + \int_{t_2}^{T_b} (x_b + y_b t)\, I_b(t)dt\right] \quad (17)$$

(iii) Deterioration Cost:

$$DC_b = nc_b\left(\int_{t_1}^{t_2} \theta I_b(t)dt + \int_{t_2}^{T_b} \theta t I_b(t)dt\right) \quad (18)$$

(iv) Sales Revenue:

$$SR_b = np\left(\int_0^{Tb} (a+bt)dt\right) = np\left(aT_b + \frac{1}{2}bT_b^2\right) \quad (19)$$

(by neglecting higher powers of θ)

(v) Total Profit:

$$TP_b = \frac{1}{T}\left[SR_b - OC_b - HC_b - DC_b\right] \quad (20)$$

Vendor's Relevant costs:

(i) Ordering Cost (OC_v) = A_v (21)

(ii) Holding Cost:

$$HC_v = x_v\left[\int_0^T I_v(t)dt - n\left\{\int_0^{T_b} I_b(t)dt\right\}\right] + y_v\left[\int_0^T tI_v(t)dt - n\left\{\int_0^{T_b} tI_b(t)dt\right\}\right]$$

$$= x_v\left[\int_0^T I_v(t)dt - n\left\{\int_0^{t_1} I_b(t)dt + \int_{t_1}^{t_2} I_b(t)dt + \int_{t_2}^{T_b} I_b(t)dt\right\}\right] + y_v\left[\int_0^T tI_v(t)dt - n\left\{\int_0^{t_1} tI_b(t)dt + \int_{t_1}^{t_2} tI_b(t)dt + \int_{t_2}^{T_b} tI_b(t)dt\right\}\right] \quad (22)$$

(iii) Preservation Technology Cost (PTC_v) = m (23)

(iv) Sales Revenue:

$$SR_v = c_b\left(\int_0^T (a + bt)dt\right) = c_b\left(aT + \frac{1}{2}bT^2\right) \quad (24)$$

(v) Total Profit:

$$TP_v = \frac{1}{T}\left[SR_v - OC_v - HC_v - PTC_v\right] \quad (25)$$

Situation I: Buyer and vendor take independent decision:

Here the buyer and vendor make decision independently. For given value of n, TP_b can be maximized by solving

$$\frac{\partial TP_b(T_b,p)}{\partial T_b} = 0, \quad \frac{\partial TP_b(T_b,p)}{\partial p} = 0, \text{ where } T_b = \frac{T}{n} \quad (26)$$

provided it satisfies the second order condition

$$\begin{bmatrix} \dfrac{\partial^2 TP_b(T_b,p)}{\partial T_b^{\,2}} & \dfrac{\partial^2 TP_b(T_b,p)}{\partial p \partial T_b} \\ \dfrac{\partial^2 TP_b(T_b,p)}{\partial T_b \partial p} & \dfrac{\partial^2 TP_b(T_b,p)}{\partial p^2} \end{bmatrix} > 0. \quad (27)$$

This solution (n,T,p) maximizes TP_v.

Then the total profit without collaboration is given by:

TP = max(TP_b + TP_v).

Situation-II: Joint decision of buyer and vendor:

Here joint decision is taken by buyer and vendor:

The optimum values of T must satisfy the following conditions which maximize total profit (TP) when buyer and vendor take joint decision.

$$\frac{\partial TP(T, p)}{\partial T} = 0, \quad \frac{\partial TP(T, p)}{\partial p} = 0, \quad \text{for T} \quad (28)$$

provided it satisfies the second order condition

$$\begin{bmatrix} \dfrac{\partial^2 TP(T,p)}{\partial T^2} & \dfrac{\partial^2 TP(T,p)}{\partial p \partial T} \\ \dfrac{\partial^2 TP(T,p)}{\partial T \partial p} & \dfrac{\partial^2 TP(T,p)}{\partial p^2} \end{bmatrix} > 0 \quad (29)$$

where total profit (TP) with collaboration is given by:

TP = TP_b + TP_v (30)

IV. NUMERICAL ILLUSTRATION

Various parameter values in appropriate units are taken for numerical illustration, A_b= 150, a = 1200, b=0.05, c_b= 40, θ=0.05, x_b = 5, y_b=0.05, A_v=2000, x_v = 3, y_v=0.03, m = 5, v_1=0.30, v_2 = 0.50. Table provides optimum independent and joint values of T, p and profits for buyer and vendor.

Table 1: Without collaboration and with collaboration optimum solution

	Independent Decision	Joint Decision
n	5	4
T	1.4842	1.3692
p	75.4156	56.2548
Buyer's Profit	43998.4064	41030.7931
Vendor's Profit	21448.9874	27374.4782
Total Profit	65447.3938	68405.2713

V. POST-OPTIMALITY ANALYSIS

Study of one parameter at a time, post-optimality results of above illustration is done here.

Table 2: Post-optimality Analysis Independent Decision

Para-meter	%	n	TP
a	+20%	5	89732.3858
	+10%	5	77136.5370
	-10%	5	54665.9553
	-20%	5	44793.4238
A_b	+20%	5	65353.8886
	+10%	5	65402.1695
	-10%	5	65488.1219
	-20%	6	65533.2292

Para-meter	%	n	TP
x_b	+20%	5	65321.2978
	+10%	5	65384.7057
	-10%	5	65508.7056
	-20%	5	65567.8173
θ	+20%	5	65432.5333
	+10%	5	65439.9651
	-10%	5	65454.7945
	-20%	5	65462.1466
A_v	+20%	6	65179.6362
	+10%	5	65312.6411
	-10%	5	65582.1466
	-20%	5	65716.8993
x_v	+20%	5	65234.9340
	+10%	5	65341.1639
	-10%	5	65553.6238
	-20%	6	65669.8301
ρ	+20%	5	57922.2640
	+10%	5	61343.8970
	-10%	5	70459.9747
	-20%	5	76722.5527

Table 3: Post-optimality Analysis Joint Decision

Para-meter	%	n	TP
a	+20%	4	92715.6077
	+10%	4	80107.7386
	-10%	4	57608.8778
	-20%	4	47719.3830
A_b	+20%	3	68334.6556
	+10%	3	68369.1543
	-10%	4	68449.3470
	-20%	4	68493.9426
x_b	+20%	4	68278.7269
	+10%	4	68341.4810
	-10%	3	68485.5188
	-20%	3	68568.8931
θ	+20%	4	68389.8757
	+10%	4	68397.5639
	-10%	3	68414.1588
	-20%	3	68424.3923

Para-meter	%	n	TP
A_v	+20%	4	68123.4973
	+10%	4	68261.8847
	-10%	3	68562.6282
	-20%	3	68728.4314
x_v	+20%	3	68215.7834
	+10%	3	68308.7532
	-10%	4	68522.5024
	-20%	4	68643.5479
ρ	+20%	4	61468.1551
	+10%	4	64595.6145
	-10%	4	73124.4112
	-20%	4	79093.9629

Based on the results of Table 2 and Table 3, we can observe about the optimal length of order cycle T*,p* and maximum total profits for independent as well as joint decisions.

There will be increase or decrease in value of parameter 'a' when parameter 'a' increase/ decrease independent or as well as jointly, however, when A_b, x_b, x_v, A_v, and θ increase/decrease then total profit decrease/increase in independent and joint decision case.

VI. CONCLUSION

The result shows that the optimal cycle time is significantly decreased and total profit significantly increased when buyers and vendor take joint decision as compared to independent decision taken by buyers and vendor.

We can also observe that the vendor's profit is increased and number of times order placed by buyer during cycle time is decreased when buyers and vendor take joint decision.

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