International Journal of Engineering Research and Technology. ISSN 0974-3154, Volume 12, Number 9 (2019), pp. 1584-1597

Tolerance Calculation for Robot Kinematic Parameters to Ensure End-Effector Errors within a Predetermined Limit Area

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Abstract

Robots, as well as other work machines, are typically designed in groups that are similar in structure and only vary in size. In the same group, robots have different accuracies. Therefore, the mechanical designs to ensure their accuracy are also independent of each other. This work takes a long time. In this paper, a method for calculating the initial accuracy for a distinct manipulator and for robots in the same structural group based on a sample robot is introduced. Theories of similarity as well as a hypothesis of the existence of the dimensional similarity ratio and accuracy ratio are used. The results show that using these ratios helps create the first approximation value of geometric tolerance easily. Because it is only necessary to calculate the manufacturing tolerance for a manipulator in a group of robots, the tolerances of other robots (even if they are not similar but are only in a structure group) are determined quickly and accurately by the theoretical application that we propose. The results of this work open up the prospect of calculating tolerances for industrial robots faster, more simply and more accurately than traditional techniques.

Keywords: Manufacturing tolerance, robot, initial accuracy, spherically permissible region, similarity, GRG method.

I. INTRODUCTION

Manipulators play an important role in industrial production. The accuracy of robots determines the quality and cost of products. The higher the accuracy, the better the product quality, but the higher the cost and vice versa. Therefore, industrial robots need reasonable accuracy. Designers spend a great deal of time and effort on determining tolerances to ensure the initial accuracy of robots.

There are several factors that affect the deviation of the position and orientation of the robot end-effector. However, kinematic parameters are the main causes: joint variables and link parameters [1].

Traditionally, the manipulator parameter tolerances are mostly selected by experience and intuition [2]. This leads to the fact that the accuracies of the designed robots are still not really reasonable. A number of studies have been carried out to find link tolerance and joint clearance, but not enough.

In this article, the authors propose a technique using the generalized reduced gradient (GRG) method to determine the link and joint tolerances from the requirement of the precision of the end-effector given. On the basis of the GRG algorithm, checking and correcting the tolerances of kinematic parameters are conducted in collaboration randomly. The example applied on a six-degree-of-freedom industrial robot is presented to prove the effectiveness of this method.

In addition, to reduce the cost of the design phase, the authors propose a method of calculating the kinematic parameters of robots with the same structural form as the robot given the initial kinematic tolerance. This is reasonable when industrial robots have a very large quantity but usually have only a few typical kinematic structures and only differ in size.

The rest of the paper is divided into the following sections. Section 2 gives an overview of the studies that have been carried out on the tolerance design for the manipulator. Section 3 outlines the basis of the proposed method to design the link and joint tolerances for a robot by the GRG method. Sections 4 and 5 demonstrate how one can design tolerances for robots having the same structure (similar or dissimilar) with a sample robot to save time and effort. The new concept of dimensional similarity ratio k and accuracy ratio k_r is given as the first approximation for the problem using a numerical method with multiple loops. The applicable example is shown in Section 6 to prove the correctness of this new method. Section 7 concludes.

II. OVERVIEW OF TOLERANCE DESIGN TECHNIQUES FOR INDUSTRIAL ROBOTS

There are several reasons for robot inaccuracy. Some of the main causes are errors due to kinematic and geometric parameters, errors in the controller, errors due to the environment, etc. In particular, errors due to the kinematic parameters (links and joints) of robots accounted for more than 70% of the total errors [3]. Numerous studies have been carried out in this direction.

Sun-Ho Kim [1] presented the problem of allocating tight tolerances with low process costs (least-cost tolerance problem). That is, the problem finds the optimal tolerance of the link and joint parameters of the robot so that the cost is minimal. A cost optimization model was established using the pseudo-Boolean program. The tolerance range of the position and direction error of the end-effector was bound. Rout and Mittal [4] used the evolutionary optimization technique to simultaneously select optimal parameters and tolerances based on the minimum cost function. The tolerances of kinematics (links and joints) and dynamics (mass, torque, etc.) are assumed to have known initial values. The variables of the cost function are the tolerances sought.

Some authors have researched and evaluated the parameter tolerances (geometrical tolerances, kinematic parameter tolerances, manufacturing tolerances, joint clearance, etc.) to identify the significant parameters that affect the end-effector deviation (the reliability of the robot). Weill and Shani [3] developed a model to assess the effect of the geometric errors of the component links on the position and orientation errors of the robot end-effector and to identify the part that has greater influence. The model was implemented on a computer program (on a SILICON-GRAPHIC computer in language C). Similarly, Liou et al. [5] determined which joint tolerance has a greater influence on the accuracy of the position and direction of the gripper by experimental design techniques based on the Taguchi method. The process is compared to the Monte Carlo simulation technique. Vukobratovic and Borovac [6] evaluated the impact of manufacturing tolerances on the accuracy of the tip position and considered the effect of each component on the position and orientation of the end-effector. Ting and co-workers[7][8] analysed and evaluated the effects of joint clearance on the position and orientation deviation of linkages and manipulators. Jeong Kim et al. [9] used the advanced first-order second moment method to determine the influence of link and joint tolerances on end-effector deviation (reliability of the robot), and the approach was verified by Monte Carlo simulation. Dao Duy Son and Kazem Abhary [10] used Taguchi's design of experiment to examine the effects of kinematic parameters on the robot's accuracy. The Taguchi method was also applied by Sheikhha and Akbarzadeh [11] to determine the impact of the parameters (link tolerance and joint clearance) on gripper accuracy. The nature of these studies has not been quantified and only identifies the trend influence of the tolerance factors (kinematics, dynamics, etc.) on end-effector accuracy.

For the optimal allocation of robot parameter tolerances, some researchers surveyed the accuracy of the end-effector in the entire workspace. Rout and Mittal [2][12][13] used Taguchi experimental optimization techniques, the evolutionary optimization technique and a Monte Carlo simulation to accomplish this task. Rao and Bhatti [14] developed a probabilistic model of the manipulator kinematics and dynamics to account for the random errors in the kinematic and dynamic parameters. Gaussian distributions are assumed for the various manipulator parameters, and the joint efforts are modelled as Markov stochastic processes. However, the process of determining the optimal tolerance range was carried out with the assumption of a deviation domain of the given component parameters.

As can be seen, these studies were conducted with assumptions on the deviation ranges of the parameters: that is, the researchers did not calculate the tolerances from the beginning. The manipulator parameter tolerances are mostly selected by experience and intuition [2][15]. According to a study by Ji et al. [16], because the given conditions are almost always insufficient, the tolerances are usually regarded as equal.

Moreover, processes are not carried out in reverse. In other words, from the requirement for the end-effector accuracy, the problem of determining the tolerances of the component parameters of the robot was not implemented. Specifically, the problem of determining the kinematic tolerances of the robot, based on a similar relationship of a series of robots having the same configuration in order to reduce the volume of computing and shorten the time, has never been found in recent studies.

This paper presents a tolerance calculation method for an entire group of robots having the same structure based on a similar relationship. The tolerances of the sample robot were calculated based on the GRG method as we presented in Ref. [17]. This technique has the advantage of being fast, accurate and applicable to different robots. However, the determination of the tolerance stopped at the level of calculation of two independent tolerance groups (link lengths d_i and a_i in the Denavit–Hartenberg (D–H) table and joint variable q_i) without considering the combined impact of these two groups concurrently. The paper did not take into account the error of the end-effector when calculating according to the forward kinematic equation, especially under the condition where the links of the robot are interchangeably assembled. This can lead to the tip position not ensuring reliability. A complete calculation of the tolerance values, taking into account the ability of interchangeability, will be discussed in detail in this paper before using a similar relationship to calculate the tolerance for the entire robot group.

III. TECHNIQUES FOR DETERMINING ROBOT TOLERANCES BASED ON THE GRG METHOD

III.I Description of end-effector accuracy

This section provides a method for evaluating the accuracy and presentation of this value as the basis for the next steps of

International Journal of Engineering Research and Technology. ISSN 0974-3154, Volume 12, Number 9 (2019), pp. 1584-1597

the paper. Pose accuracy is defined as the ability of the robot to precisely move to the desired position in three-dimensional space [13]. Accuracy is usually characterized by the distance from the desired point to the actual position that the structure achieves. Thus, in three-dimensional space, the accuracy of the end-effector can be described by a sphere of error control (a spherically permissible region) in which the centre of the sphere is in the desired position and the radius is equal to the enabled deviation of the tip position. The accuracy of a robot is considered to be satisfactory if at every desired position in the robot workspace the actual approach point is within this sphere (see Fig. 1).

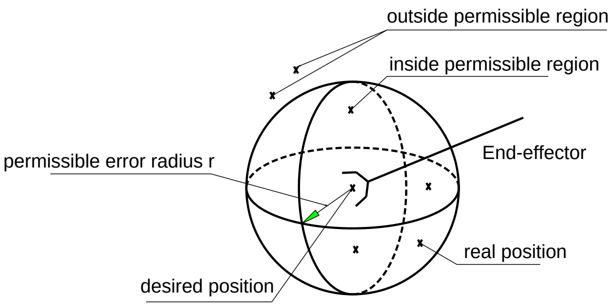

Fig. 1. Accuracy of the end-effector position.

This description is used throughout this article. On the basis of the accuracy of the end-effector position required, the initial link and joint tolerances of the robot are determined. At the same time, the relationship between the tip positioning accuracy r and the link tolerances is set. The process of checking and adjusting the tolerances in the reverse direction is also carried out based on point statistics outside the exact position allowed (outside the sphere)/total tests when the component links of a robot are interchangeable to assess the accuracy of the robot.

III.II Basis for determining the tolerances of a robot – sample robot A

Without considering the elastic deformation of the links, kinematic modelling gives the correct model as follows:

$$f(a_i, d_i, q_i) = p, \qquad i = \overline{1,n} \tag{1}$$

where n: number of degrees of freedom of the given robot;

ai: link lengths; di: link offset;

qi: generalized coordinate variables of joints;

p: position and orientation of the end-effector.

Let r be the radius of the spherically permissible region of the end-effector at p. The kinematic model of robots when fully considering the link and joint errors is given below:

$$f(a_i \pm \delta a_i, d_i \pm \delta d_i, q_i \pm \delta q_i) = p \pm d\mathrm{r}, \qquad i = \overline{1,n} \tag{2}$$

However, the problem determining link tolerances and joint tolerances concurrently is difficult and complicated. The problem is divided into two more simple ones, as shown in Eqs. (3) and (4). Then, the tolerances of the link parameter group d_i, a_i and the generalized coordinate group q_i were separately determined without considering their effects on each other.

$$f(a_i \pm \delta a_i, d_i \pm \delta d_i, q_i) = p \pm \frac{dr}{2}, \qquad i = \overline{1,n} \tag{3}$$

where $\frac{dr}{2}$ denotes the radius of the spherically permissible region of the end-effector at p due to the impact of the component link errors. From this equation, $\delta a_i, \delta d_i$ are computed when q_i = const.

By contrast, to determine the generalized coordinate tolerance, Eq. (4) below is used:

$$f(a_i, d_i, q_i \pm \delta q_i) = p \pm \frac{dr}{2}, \qquad i = \overline{1,n} \tag{4}$$

In this equation, a_i and d_i are nominal dimensions without tolerance.

$\frac{dr}{2}$ denotes the radius of the spherically permissible region of the end-effector at p due to the impact of the joint errors.

On the basis of the GRG method, the link and joint tolerance values $\delta a_i, \delta d_i, \delta q_i$ defined through Eqs. (3) and (4) are the first approximation values. To find exactly the set of tolerances that satisfy the problem requirement, it is necessary to check and correct the errors based on the forward kinematic problem. A specialized software developed by the research team will perform this task to scan all cases that can occur on the entire tolerance of component links and joints.

Let us denote e_i as the error at a survey point of the end-effector, defined as follows:

$$e_i = f(a_i \pm \delta a_i, d_i \pm \delta d_i, q_i \pm \delta q_i) - f(a_i, d_i, q_i), \qquad i = \overline{1,n} \tag{5}$$

The tolerance problem ends when

$$e_i \le d\mathrm{r} \qquad \forall i = \overline{1,n} \tag{6}$$

If Eq. (6) is not satisfied, the width of one of the three values $\delta a_i, \delta d_i, \delta q_i$ must be corrected following Fig. 2.

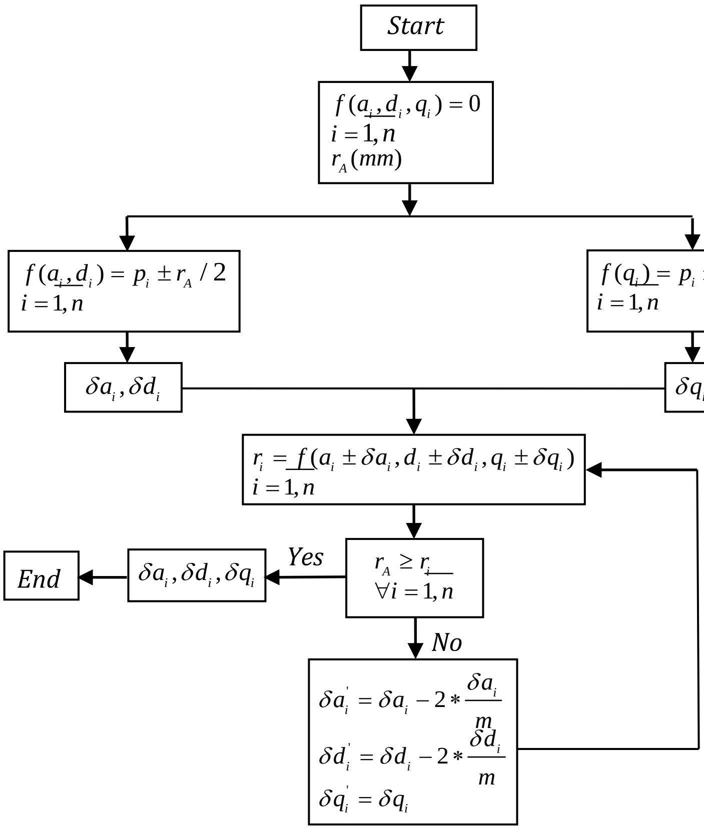

Fig. 2. Chart for correcting tolerances with combined checking.

According to the manufacturer's standard, sensors and motors have standard resolutions that are quite distant from each other, and refining a small amount of δq_i near the standard value is very difficult to implement. Thus, if δq_i is selected, to satisfy inequality (6), $\delta a_i, \delta d_i$ are corrected by decreasing their values step by step to a sufficient degree Eq. (6). Divide the intervals $[a_i - \delta a_i^{min}, a_i + \delta a_i^{max}], [d_i - \delta d_i^{min}, d_i + \delta d_i^{max}]$ into equal parts m that are sufficiently small. When the relationship in Eq. (4) is unsatisfactory, it is necessary to reduce the length of the parts by one division at each end and perform a reexamination. That is, in the *j*th test step, the width of the divisions is

$$\frac{[\delta a_i^{max} - \delta a_i^{min}]}{m} * (m-2j) \quad and \quad \frac{[\delta d_i^{max} - \delta d_i^{min}]}{m} * (m-2j) \tag{7}$$

If this interval satisfies Eq. (4), it will be the final value recorded on the fabrication drawings of the corresponding links.

In the case of automatic adjustment of these tolerances, one or more link tolerances are zero or too tight, and this is unrealistic in manufacturing. This problem is overcome by reducing the link tolerance having the loosest tolerance value or by reducing looser tolerances in advance. When the tolerances are relatively uniform, the adjustment process will be performed simultaneously to maintain economy in manufacturing.

After completing the design of the link and joint tolerances for a robot, the design of tolerances for a group of robots having the same structure is presented in Sections 4 and 5: design tolerance for a robot similar to the sample robot and design tolerance for robots of the same structure type but not similar to the sample robot.

IV. PROBLEM IDENTIFYING TOLERANCES FROM A SIMILAR SAMPLE ROBOT

IV.I Concepts

Although industrial manipulators have a large quantity, there are usually only a few typical kinematic structures. Manufacturers have produced several generations of robots: they only differ in size, but their structures are constant. It is possible to name the major manufacturers that have created various industrial robots based on this typical structure: Kuka, Fanuc, Kawasaki, ABB, etc. The kinematic model in Fig. 3 is a structure like this: a robot structure with six degrees of freedom with the combination of biomimetic structure (03 first joints) and spherical structure (03 rear joints).

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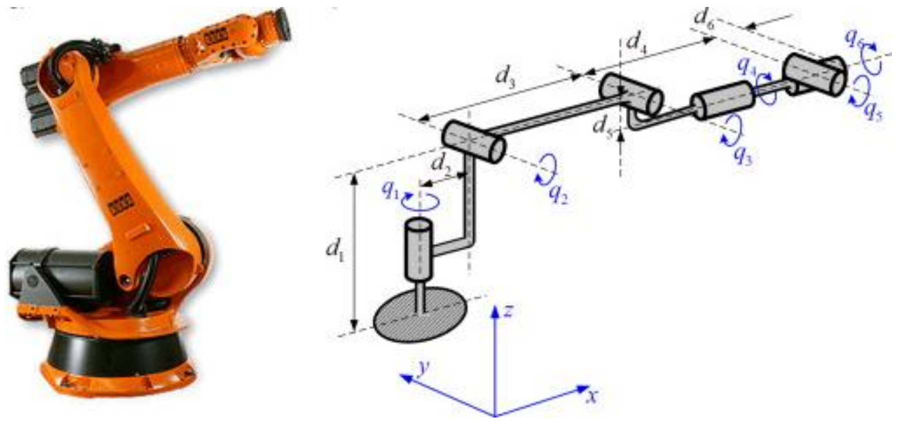


Fig. 3. Kinematic diagram of a serial robot with six degrees of freedom.

In one family, the robots have the same structure, but the dimensions are usually proportional to each other according to the design theory. On a robot arm, the ratio of the length between the links is determined. Therefore, the idea of the problem of calculating the tolerance of kinematic parameters of the manipulator based on a similar relationship is given.

For that purpose, the following concepts are introduced:

- Dimensional similarity ratio: the ratio of the length of two links of two robots in a similar group (see Fig. 4).

For example, if A and B are two similar robots, the dimensional similarity ratio is defined as

$$k = \frac{d_1^{(A)}}{d_1^{(B)}},\; k = \frac{d_2^{(A)}}{d_2^{(B)}}, \qquad (8)$$

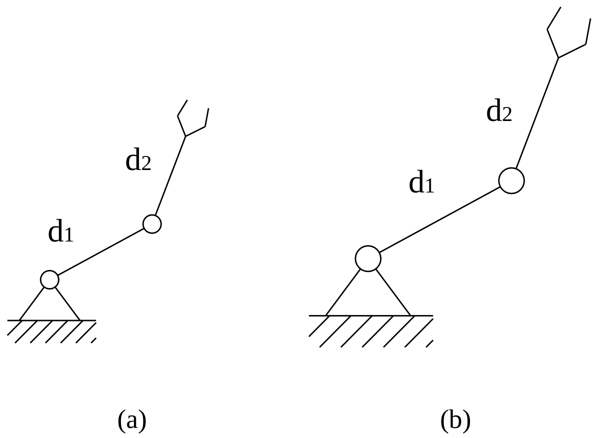


Fig. 4. Description of two similar robots: (a) robot A; (b) robot B.

- Accuracy ratio:

Two robots A and B have end-effector errors not exceeding spheres (circles) whose radii are r_A and r_B, respectively (see Fig. 5). The accuracy ratio of two robots is evaluated by

$$k_r = \frac{r_A}{r_B} \qquad (9)$$

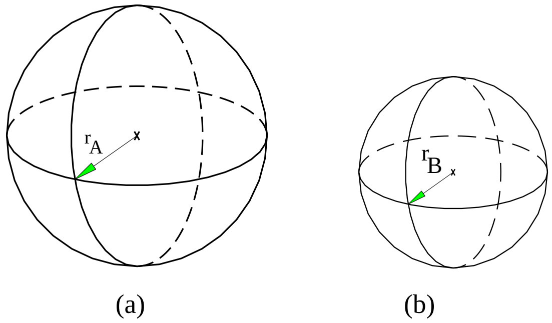


Fig. 5. Description of the permissible accuracy of (a) robot A and (b) robot B.

- Two similar robots:

If two robots have the same structure and the links relate according to the k-ratio, they are considered to be in the same class. Geometric similarity between them occurs when they are in a pose having the same set of generalized coordinates (q_i).

The ratios k, k_r and the relative relationship between them create a premise to bind the end-effector accuracy with the robot kinematic parameter tolerances. That helps to determine the most reasonable first approximation when solving numerical problems with many loops according to Fig. 6.

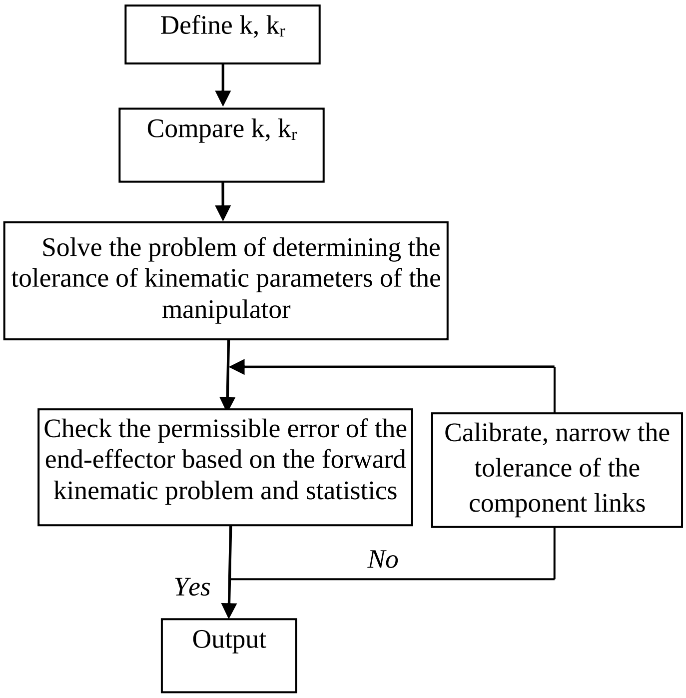


Fig. 6. Chart for determining the kinematic parameter tolerances of a similar robot.

IV.II Basic problems

- Problem 1: Two robots are similar, and the dimensional similarity ratio equals the accuracy ratio ($k_r = k$).
- Problem 2: Two robots are similar, and the dimensional similarity ratio differs from the accuracy ratio ($k_r \neq k$).

Suppose that two robots have similarly nominal dimensions as A and B. Robot A has the end-effector accuracy and the kinematic tolerances given. At the *i*th survey posture, A and B have the same generalized coordinates $(q_1, \ldots, q_n)(iA,B)$ and D–H dimension tolerances $(\delta a_1,..,\delta a_n;\delta d_1,..,\delta d_n)^{(iA)}$ and $(\delta a_1,..,\delta a_n;\delta d_1,..,\delta d_n)^{(iB)}$, respectively. A similar equation follows:

$$\left(a_1,..,a_n;d_1,..,d_n\right)^{(A)} = k(a_1,..,a_n;d_1,..,d_n)^{(B)} \quad (10)$$

Because the two robots satisfy the similarity, it is always possible to establish a derivative relationship of Eq. (10) for the *i*th survey point of the workspace as follows:

$$(\delta a_1,..,\delta a_n;\delta d_1,..,\delta d_n)^{(iA)} = k(\delta a_1,..,\delta a_n;\delta d_1,..,\delta d_n)^{(iB)} \quad (11)$$

Thus, if A and B are geometrically similar with ratio k and the tolerance bands of the D–H parameters of A are known, it is easy to determine the tolerance limits of the parameters of the remaining robot B based on a similar relationship.

In problem 1, the dimensional similarity ratio equals the accuracy ratio. This case only needs to know the tolerance of either robot A or B and calculates the remaining robot according to Eq. (11).

$$k_r = k$$

In problem 2, the two robots are similar, but the dimensional similarity ratio differs from the accuracy ratio ($k_r \neq k$). That is, the accuracy is not scaled like the dimensional ratio of the robot. A tolerance similarity model is applied as below.

Call A the sample robot with the end-effector accuracy r_A and the tolerances of the links determined according to Section 3.2. Robot B' is similar to A in proportion k. The end-effector accuracy needs to be achieved to be r_B'. In this case,

$$k = \frac{d_i^{(A)}}{d_i^{(B')}} \neq \frac{r_A}{r_{B'}} \quad \text{or} \quad k \neq k_r$$

To find the tolerance of the component links of B′

Call B the intermediate robot –B with the configuration of B', that is, $k = \frac{d_i^{(B)}}{d_i^{(B')}} = 1$, and compare it with the sample robot A: $k = \frac{d_i^{(A)}}{d_i^{(B)}} = \frac{r_A}{r_B}$, that is, $k = k_r$.

Thus, the tolerance of the intermediate robot B is found according to problem 1. Determine the tolerance of robot B' according to B. That is, know the link tolerances of B and the accuracy of the end-effector of r_B, $r_{B'}$.

To determine the component link tolerance of B′

For robot B, the link tolerances identified from A are $(\delta a_1,..,\delta a_n;\delta d_1,..,\delta d_n)^{(B)}$. The establishment of component relationships between the permissible error of the end-effector and the tolerances of the component links according to the rates is as follows:

$$\left(\frac{r_B}{\delta a_1},..,\frac{r_B}{\delta a_n};\frac{r_B}{\delta d_1},..,\frac{r_B}{\delta d_n}\right)^{(B)} \quad (12)$$

Assume that this is also true for robot B′. It is possible to apply the above ratio to distribute the permissible error of the end-effector r'_B for the tolerances of the component links $(\delta a_1,..,\delta a_n;\delta d_1,..,\delta d_n)^{(B')}$ in order to accelerate the computational speed.

Let m be the common divisor of r_B and r'_B and

$$i_1 = \frac{r_B}{m},\ i_2 = \frac{r'_B}{m} \quad (13)$$

Then, the relationship of the unit displacement between the joint space and the workspace of robot B is assumed to be

$$\frac{r_B}{i_1} \rightarrow \left(\frac{\delta a_1^{(B)}}{i_1},..,\frac{\delta a_n^{(B)}}{i_1};\frac{\delta d_1^{(B)}}{i_1},..,\frac{\delta d_n^{(B)}}{i_1}\right) \quad (14)$$

If this ratio is applied to robot B′ based on similarity, the tolerances of the corresponding links will be

$$\left(\frac{\delta a_1^{(B)}}{i_1}*i_2,..,\frac{\delta a_n^{(B)}}{i_1}*i_2;\frac{\delta d_1^{(B)}}{i_1}*i_2,..,\frac{\delta d_n^{(B)}}{i_1}*i_2\right)^{(B')} \quad (15)$$

The reallocation of the tolerances of the joint variables of B′ also uses the above rule:

$$\left(\frac{\delta q_1^{(B)}}{i_1}*i_2,..,\frac{\delta q_2^{(B)}}{i_1}*i_2,..,\frac{\delta q_n^{(B)}}{i_1}*i_2\right)^{(B')} \quad (16)$$

Thus, this method saves time because it does not have to recalculate the tolerance of robot B′ from the beginning. The ratio distributing the upper and lower deviations of the tolerance of B′ is taken from that of B as a sample.

Note that the values in Eqs. (15) and (16) are the first approximation values, and they need to be checked to determine reasonable values by using the kinematic equation based on the method of exhaustion.

V. PROBLEM DETERMINING THE TOLERANCE FROM TWO ROBOTS WITH THE SAME STRUCTURE BUT NOT SIMILAR (PROBLEM 3)

If a robot is designed for a certain purpose and is not similar but is still in the same structure group as a calculated prototype robot, the computation can be performed as in this section (see Fig. 7).

Assume that robot A has the following kinematic parameters:

- link parameters $a_i^A, d_i^A,\ i=\overline{1,n}$ with corresponding tolerances $\delta a_i^A, \delta d_i^A,\ i=\overline{1,n}$,
- joint variables q_i, $i=\overline{1,n}$ with corresponding clearances δq_i (rad), and
- a spherical permissible region with radius r_A (mm).

The results of the link and joint tolerances of robot A are calculated separately in two independent problems by the GRG method as presented above. Let D be a robot in the same group, but not similar to A, that is, the ratio between their respective links is not equal:

$$\frac{d_i^D}{d_i^A} \neq \frac{d_{i+1}^D}{d_{i+1}^A}$$

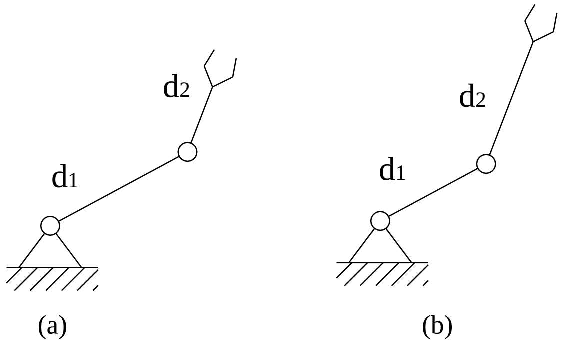


Fig. 7. Description of two robots belonging to the same structure group but not similar: (a) robot A; (b) robot D.

The positioning accuracies of the end-effectors r_A and r_D are different. The kinematic tolerance of robot D is found through robot A and intermediate robot C in two phases, as shown below.

Phase 1 *(Find r_C for intermediate robot C)*

Let C be a robot with a similar structure to robot D and in the same structure group but not similar to robot A, that is,

$$\frac{d_i^C}{d_i^A} \neq \frac{d_{i+1}^C}{d_{i+1}^A}$$

With the GRG technique applied to robot A, the reasonable ratio between the tolerance and the corresponding link length of this type of robot structure was determined as $\delta d_i^A / d_i^A$. The natural kinematics of the robot according to this ratio can be considered to be reasonable. Without loss of generality, suppose that robot C in the same group A uses the same ratio for link consideration, that is, it has the following relationship:

$$\frac{\delta d_i^A}{d_i^A} = \frac{\delta d_i^C}{d_i^C} \rightarrow \delta d_i^C = \delta d_i^A * \frac{d_i^C}{d_i^A} \quad (17)$$

When d_i^C changes its nominal length compared with d_i^A, its tolerance is recalculated by this ratio.

After the tolerance of links δd_i^C is defined as Eq. (17), r_C is found by using the forward kinematic equation to sweep the combinations of C. The joint tolerances δq_i^C of C stay the same as those of robot A. The parameters are passed to the computational software to determine the radius of the spherically permissible region for all combinations and to find the maximum possible spherical radius $r_C = \max(r_{Ci})$.

The parameter used for phase 2 is r_C; therefore, after determining this value, phase 1 ends.

Phase 2 *(Find the tolerance of robot D based on robot C being similar to D)*

Robot D is in the same group as sample robot A and has an identical configuration as the intermediate robot C. We need to find the kinematic parameter tolerances of D according to the requirement from the end-effector accuracy r_D.

Let D be a robot with the same configuration as C, but $r_C \neq r_D$.

Let m be the common divisor of r_C and r_D, which states that

$$i_1 = \frac{r_C}{m}, i_2 = \frac{r_D}{m} \quad (18)$$

Assume that the relationship of the unit displacement between the joint space and the workspace of robot C is

$$\frac{r_C}{i_1} \rightarrow \left(\frac{\delta a_1^{(C)}}{i_1},..,\frac{\delta a_n^{(C)}}{i_1};\frac{\delta d_1^{(C)}}{i_1},..,\frac{\delta d_n^{(C)}}{i_1}\right) \quad (19)$$

If this ratio is applied to robot D on a similar basis, the tolerances of the corresponding links will be

$$\left(\frac{\delta a_1^{(C)}}{i_1}*i_2,..,\frac{\delta a_1^{(C)}}{i_1}*i_2;\frac{\delta d_1^{(C)}}{i_1}*i_2,..,\frac{\delta d_n^{(C)}}{i_1}*i_2\right)^{(D)} \quad (20)$$

The reallocation of the coordinates' generalized tolerance of D also uses the above rule:

$$\left(\frac{\delta q_1^{(C)}}{i_1}*i_2,..,\frac{\delta q_2^{(C)}}{i_1}*i_2,..,\frac{\delta q_n^{(C)}}{i_1}*i_2\right)^{(D)} \quad (21)$$

By combining the above two techniques, it is possible to determine the parameter tolerances of a robot in the same group D but not similar to sample robot A with a precalculated deviation. This always results in faster calculation than when determining the tolerances for robot D according to the standard procedure from the beginning.

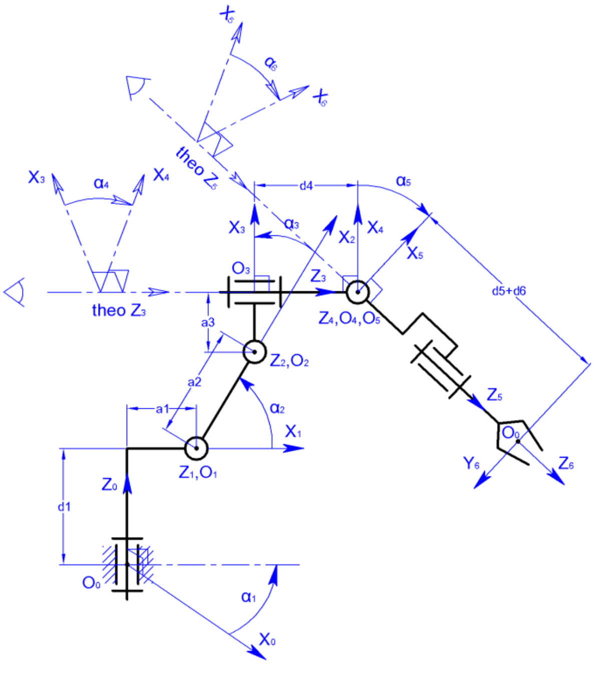
Note that these are the first approximation values, and they need to be checked to determine reasonable values on the software based on the method of exhaustion.

VI. ILLUSTRATIVE CALCULATION WITH A SIX-DEGREE-OF-FREEDOM ROBOT

The illustration process is carried out with the following basic steps. First, a robot with six degrees of freedom is calculated, and the tolerance based on the given configuration and the required end-effector accuracy is determined. Next, according to the robot that has calculated the tolerance, the tolerances of other robots having the same structural form are defined. This section is divided into two cases: (1) The robot needed to calculate the tolerance is similar to the sample robot, that is, two robots have the same structure but only differ in size. (2) The robot needed to calculate the tolerance has the same structural form as the sample robot but is not similar, that is, two robots have the same kinematic model, but the size ratio of the component links is different.

VI.I Problem 1: Calculate the kinematic parameter tolerances of a robot –sample robot A

Consider sample robot A with the kinematic model as displayed in Fig. 8.



(a)

	R_z	T_z	T_x	R_x
1	(α_1)	d_1	a_1	90^0
2	(α_2)	0	a_2	0
3	(α_3)	0	a_3	90^0
4	(α_4)	d_4	0	-90^0
5	(α_5)	0	0	90^0
6	(α_6)	d_6	0	0

(b)

Fig. 8. Robot A and kinematic parameters: (a) typical six-rotary-joint robot; (b) D–H table.

The specific dimensions of robot A are assigned as shown in Table 1.

Table 1: Link dimensions of robot A

d_1 (mm)	a_1 (mm)	a_2 (mm)	a_3 (mm)	d_4 (mm)	d_6 (mm)
335	75	270	90	295	80

The end-effector accuracy is assumed not to exceed a radius of R = 1 mm of the spherically permissible region. On the basis of the GRG method, separate solutions to the problems of finding the joint angle and the link length tolerances [17] are shown in Table 2 (in which the joint variable tolerances were chosen according to the standard resolution of sensors and motors mounted on those joints).

Table 2: Kinematic parameter tolerances of robot A

$\delta q_1, \delta q_2, \delta q_3$ (rad)	$\delta q_4, \delta q_5, \delta q_6$ (rad)	δd1 (mm)	δa1 (mm)	δa2 (mm)	δa3 (mm)	δd4 (mm)	δd6 (mm)
±0.000393	±0.000314	±0.150	±0.100	±0.150	±0.100	±0.100	±0.100

Specialized software is built based on forward kinematic relationships with D–H representation to check the position of the end-effector in the workplace. The software also has the function of calibrating each link and joint tolerance as desired and checking again.

In all, 531,441 cases of random combinations of link lengths and joint angles within the calculated tolerance range were tested. The results show that there is no point outside the spherically permissible region with radius r = 1 mm. Visual images are simulated by the model of Fig. 9.

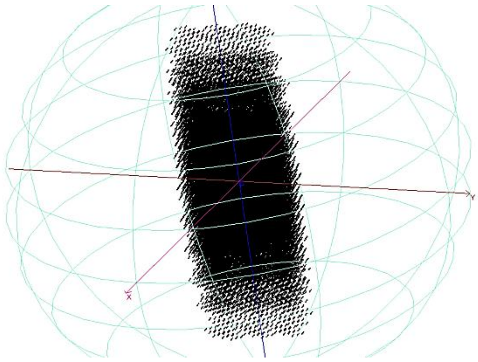


Fig. 9. Simulation of robot's end-effector coordinates in the spherically permissible region R = 1mm.

Thus, for a robot with the required end-effector accuracy, based on solving the reverse kinematic problem according to the GRG method, a set of values for link and joint tolerances is found. The test results following the forward kinematic problem for 531,441 cases of interchangeable component link and joint parameters showed that the tolerance of the kinematic parameters found above is satisfactory.

VI.II Problem 2: Calculate the kinematic parameter tolerances of robot B similar to robot A

Robot B is similar to robot A. That is, the kinematic models of A and B are similar, and they have the same ratio of component link lengths. B has the parameters given in Table 3.

Table 3: Link dimensions of robot B

d_1 (mm)	a_1 (mm)	a_2 (mm)	a_3 (mm)	d_4 (mm)	d_6 (mm)
167.5	37.5	135.0	45.0	147.5	40.0

On the basis of the correlation between the dimensional similarity ratio k and the accuracy ratio k_r, consider the following cases.

VI.II.I Robots B and A satisfy k = k_r

The radius of the error sphere at the end-effector is $r_B = 0.5$ mm, that is, k = k_r = 2.

According to the theory presented above, the tolerances of robot B (see Table 4) are defined as

$$\delta d_1^B = \frac{\delta d_1^A}{2} = \frac{0.3}{2} = 0.15\,\text{mm}, \quad \delta a_1^B = \frac{\delta a_1^A}{2} = \frac{0.2}{2} = 0.1\,\text{mm}$$

$$\delta a_2^B = \frac{\delta a_2^A}{2} = \frac{0.3}{2} = 0.15\,\text{mm}, \quad \delta a_3^B = \frac{\delta a_3^A}{2} = \frac{0.2}{2} = 0.1\,\text{mm}$$

$$\delta d_4^B = \frac{\delta d_4^A}{2} = \frac{0.2}{2} = 0.1\,\text{mm}, \quad \delta d_6^B = \frac{\delta d_6^A}{2} = \frac{0.2}{2} = 0.1\,\text{mm}$$

We distribute the deviation on two sides according to the rules of robot A.

Table 4: Kinematic parameter tolerances of robot B

δd_1 (mm)	δa_1 (mm)	δa_2 (mm)	δa_3 (mm)	δd_4 (mm)	δd_6 (mm)
±0.075	±0.050	±0.075	±0.050	±0.050	±0.050

The calculated data are input into the software to test the forward direction at some positions in the robot workspace. The results indicated that all point combinations generated during the test fall within the sphere with a radius of 0.5 mm. Thus, the calculation of link length tolerance for robot B is completely reasonable and reliable.

VI.II.II Robots B′ and A with k≠ k_r

B′ has the same kinematic parameters as robot B, but the required accuracy of the tip position of B′ is different from that of robot B, that is, $r'_B \neq r_B$.

- Case 1: $r^{B'} < r^B$ (B''s end-effector accuracy > B's end-effector accuracy)

Suppose that $r^{B'} = 0.3$ mm, with the common divisor m = 0.05.

$$i_1 = \frac{r^B}{0.05} = \frac{0.5}{0.05} = 10, \quad i_2 = \frac{r^{B'}}{0.05} = \frac{0.3}{0.05} = 6$$

Because of $\frac{r_B}{10}=\frac{0.5}{10}$, the unitary influence of robot B is $\frac{\delta a_i^{(B)}}{10}$

The results for calculating the tolerances of robot B′ (see Table 5) are

$$(\frac{\delta d_1^{(B)}}{i_1}*i_2,\frac{\delta a_1^{(B)}}{i_1}*i_2,\frac{\delta a_2^{(B)}}{i_1}*i_2,\frac{\delta a_3^{(B)}}{i_1}*i_2,\frac{\delta d_4^{(B)}}{i_1}*i_2,\frac{\delta d_6^{(B)}}{i_1}*i_2)$$

$$(\frac{\delta q_1^{(B)}}{i_1}*i_2,\frac{\delta q_2^{(B)}}{i_1}*i_2,\frac{\delta q_3^{(B)}}{i_1}*i_2,\frac{\delta q_4^{(B)}}{i_1}*i_2,\frac{\delta q_5^{(B)}}{i_1}*i_2,\frac{\delta q_6^{(B)}}{i_1}*i_2)$$

The details are as follows:

$$\delta d_1^{B'}=\frac{0.15}{10}*6=0.09,\quad \delta q_1^{B'}=\frac{0.000393}{10}*6=0.000236$$

$$\delta a_1^{B'}=\frac{0.1}{10}*6=0.06,\quad \delta q_2^{B'}=\frac{0.000393}{10}*6=0.000236$$

$$\delta a_2^{B'}=\frac{0.15}{10}*6=0.09,\quad \delta q_3^{B'}=\frac{0.000393}{10}*6=0.000236$$

$$\delta a_3^{B'}=\frac{0.1}{10}*6=0.06,\quad \delta q_4^{B'}=\frac{0.000314}{10}*6=0.000188$$

$$\delta d_4^{B'}=\frac{0.1}{10}*6=0.06,\quad \delta q_5^{B'}=\frac{0.000314}{10}*6=0.000188$$

$$\delta d_6^{B'}=\frac{0.1}{10}*6=0.06,\quad \delta q_6^{B'}=\frac{0.000314}{10}*6=0.000188$$

We redistribute these tolerances with the upper and lower deviations according to the sample ratio of robot B.

Table 5: Kinematic parameter tolerances of robot B′

δq1, δq2, δq3 (rad)	δq4, δq5, δq6 (rad)	δd1 (mm)	δa1 (mm)	δa2 (mm)	δa3 (mm)	δd4 (mm)	δd6 (mm)
±0.000118	±0.000094	±0.045	±0.030	±0.045	±0.030	±0.030	±0.030

The test interchanging the components of the manipulator on the software receives the results: all 531,441 survey points are in the error control sphere. Hence, the tolerance values of B' are satisfactory.

- Case 2: $r^{B'}>r^{B}$ (B''s end-effector accuracy < B's end-effector accuracy)

Suppose $r^{B'}=0.7$ mm. Only the tolerances of the links are recalculated, and the generalized variable tolerances are retained. In this case, the common divisor m = 0.1.

$$i_1=\frac{r^B}{0.05}=\frac{0.5}{0.1}=5,\qquad i_2=\frac{r^{B'}}{0.1}=\frac{0.7}{0.1}=7$$

The results for calculating the tolerances for robot B′ (see Table 6) are

$$(\frac{\delta d_1^{(B)}}{i_1}*i_2,\frac{\delta a_1^{(B)}}{i_1}*i_2,\frac{\delta a_2^{(B)}}{i_1}*i_2,\frac{\delta a_3^{(B)}}{i_1}*i_2,\frac{\delta d_4^{(B)}}{i_1}*i_2,\frac{\delta d_6^{(B)}}{i_1}*i_2)$$

This means that

$$\delta d_1^{B'}=\frac{0.15}{5}*7=0.21\text{ mm},\quad \delta a_1^{B'}=\frac{0.1}{5}*7=0.14\text{ mm}$$

$$\delta a_2^{B'}=\frac{0.15}{5}*7=0.21\text{ mm},\quad \delta a_3^{B'}=\frac{0.1}{5}*7=0.14\text{ mm}$$

$$\delta d_4^{B'}=\frac{0.1}{5}*7=0.14\text{ mm},\quad \delta d_6^{B'}=\frac{0.1}{5}*7=0.14\text{ mm}$$

We redistribute these tolerances with the upper and lower deviations according to the sample ratio of robot B.

Table 6: Kinematic parameter tolerances of robot B′

δd1(mm)	δa1(mm)	δa2(mm)	δa3(mm)	δd4(mm)	δd6 (mm)
±0.105	±0.070	±0.105	±0.070	±0.070	±0.070

The generalized variable tolerances remain like those of robot B (see Table 7).

Table 7: Joint variable tolerances of robot B′

δq1, δq2, δq3 (rad)	δq4, δq5, δq6 (rad)
±0.000393	±0.000314

The test results fully satisfy the end-effector accuracy when the links and joints are interchangeable. Thus, the tolerances of link lengths and joint angles as calculated are correct and reasonable.

VI.III Problem 3: Computing the kinematic tolerances of robot D –the same structure but not similar to robot A

Assume that robot D has the nominal kinematic parameters shown in Table 8.

Table 8: Link dimensions of robot D

d_1^D (mm)	a_1^D (mm)	a_2^D (mm)	a_3^D (mm)	d_4^D (mm)	d_6^D (mm)
250	65	220	80	200	60

It can be seen that D and A have the same kinematic model, but the ratio of link length between them is different. The radius of the spherically permissible region is $r_D=0.5$ mm. The two-phase theory (Section 5) based on sample robot A is used.

- ***Phase 1: Determining the radius r_c of intermediate robot C***

Let C be an intermediate robot in the same structural form but not similar to robot A, and let C have kinematic dimensions similar to those of robot D (see Table 9), that is,

$$(\frac{d_i^{\,C}}{d_i^{\,A}}\neq\frac{d_{i+1}^{\,C}}{d_{i+1}^{\,A}})\text{ and } r_C\neq r_A$$

International Journal of Engineering Research and Technology. ISSN 0974-3154, Volume 12, Number 9 (2019), pp. 1584-1597

Table 9: Link dimensions of robot C

d_1^C (mm)	a_1^C (mm)	a_2^C (mm)	a_3^C (mm)	d_4^C (mm)	d_6^C (mm)
250	65	220	80	200	60

We confirm the dimensional tolerances of robot C (using Eq. (17)):

$$\delta d_1^C = \delta d_1^A * \frac{d_1^C}{d^A} = 0.3*\frac{250}{335} = 0.224 \leftrightarrow \delta d_1^C = \pm 0.112 \text{ (mm)}$$

$$\delta a_1^C = \delta a_1^A * \frac{a_1^C}{a_1^A} = 0.2*\frac{65}{75} = 0.173 \quad \leftrightarrow \quad \delta a_1^C = \pm 0.087 \text{ (mm)}$$

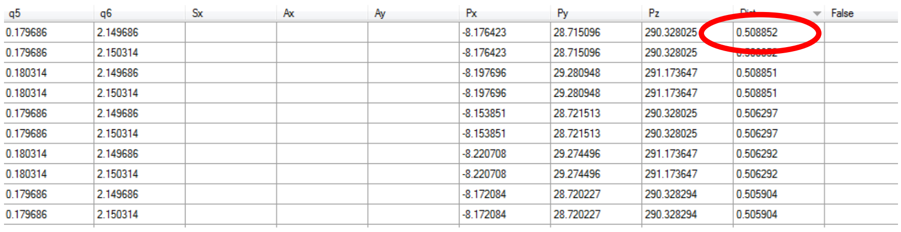
$$\delta a_2^C = \delta a_2^A * \frac{a_2^C}{a_2^A} = 0.3*\frac{220}{270} = 0.244 \quad \leftrightarrow \quad \delta a_2^C = \pm 0.122 \text{ (mm)}$$

$$\delta a_3^C = \delta a_3^A * \frac{a_3^C}{a_3^A} = 0.2*\frac{80}{90} = 0.178 \quad \leftrightarrow \quad \delta a_3^C = \pm 0.089 \text{ (mm)}$$

$$\delta d_4^C = \delta d_4^A * \frac{d_4^C}{d_4^A} = 0.2*\frac{200}{295} = 0.136 \leftrightarrow \delta d_4^C = \pm 0.068 \text{ (mm)}$$

$$\delta d_6^C = \delta d_6^A * \frac{d_6^C}{d_6^A} = 0.2*\frac{60}{80} = 0.150 \leftrightarrow \delta d_6^C = \pm 0.075 \text{ (mm)}$$

To determine the radius r_C, it is necessary to investigate the forward kinematic problem at locations in the robot's workspace. The computational software is used to scan all possible combinations when surveying.



q5	q6	Sx	Ax	Ay	Px	Py	Pz	Dist	False
0.179686	2.149686				-8.176423	28.715096	290.328025	0.508852	
0.179686	2.150314				-8.176423	28.715096	290.328025	[illegible]	
0.180314	2.149686				-8.197696	29.280948	291.173647	0.508851	
0.180314	2.150314				-8.197696	29.280948	291.173647	0.508851	
0.179686	2.149686				-8.153851	28.721513	290.328025	0.506297	
0.179686	2.150314				-8.153851	28.721513	290.328025	0.506297	
0.180314	2.149686				-8.220708	29.274496	291.173647	0.506292	
0.180314	2.150314				-8.220708	29.274496	291.173647	0.506292	
0.179686	2.149686				-8.172084	28.720227	290.328294	0.505904	
0.179686	2.150314				-8.172084	28.720227	290.328294	0.505904	

Fig. 10. Test results of end-effector accuracy at a survey position in software.

As shown in Fig. 10, there are 4096 radii r_C calculated at this survey point, and the radius with the largest value is defined as max {dist} = 0.508852 mm. A similar calculation is performed for 10 other points in the workspace. The results are shown in Table 10.

Table 10: Statistics of radius r_C of robot C for 10 survey points

	q1	*q2*	*q3*	*q4*	*q5*	*q6*	*Dist (max)*
1	1.5700	1.3400	1.7500	0.870	0.18	2.150	0.508852
2	0.1700	0.3400	0.7500	0.370	0.20	0.150	0.616425
3	0.1700	0.1400	0.2500	0.370	0.20	0.550	0.647996
4	0.34567	0.1400	0.6440	0.370	0.20	0.550	0.606309
5	0.34567	0.5434	0.3440	0.370	0.64	0.074	0.633535
6	0.34567	0.65434	0.3444	1.370	0.14	1.074	0.631045
7	0.1567	0.05434	0.1434	0.120	0.14	0.074	0.663631
8	0.1567	0.15434	0.14344	0.320	0.14	0.074	0.656459
9	0.0000	0.0000	0.0000	0.000	0.00	0.000	0.674742
10	0.0760	0.0000	1.0000	0.045	0.76	0.000	0.569638
Max (dist) = r_C							**0.674742**

After examining the points in the workspace of robot C with the calculated tolerances, the largest r_C of the survey positions was r_C = max {dist} = 0.674742 mm, and r_C = 0.7 mm was chosen.

- ***Phase 2: Finding the tolerances of robot D based on robot C being similar to D***

Robot D is a robot with the same configuration as C, but $r_C \neq r_D$.

$$r_C = 0.7 \text{ mm}, \qquad r_D = 0.5 \text{ mm}$$

The common divisor m = 0.1.

$$i_1 = \frac{r^C}{0.1} = \frac{0.7}{0.1} = 7, \qquad i_2 = \frac{r^D}{0.1} = \frac{0.5}{0.1} = 5$$

Because of $\frac{r_C}{7} = \frac{0.7}{7}$, the unitary influence of robot C is $\frac{\delta d_i^{(C)}}{7}$

The results of calculating the tolerances for robot D are

$$\left(\frac{\delta d_1^{(C)}}{i_1}*i_2, \frac{\delta a_1^{(C)}}{i_1}*i_2, \frac{\delta a_2^{(C)}}{i_1}*i_2, \frac{\delta a_3^{(C)}}{i_1}*i_2, \frac{\delta d_4^{(C)}}{i_1}*i_2, \frac{\delta d_6^{(C)}}{i_1}*i_2\right)$$

$$\left(\frac{\delta q_1^{(C)}}{i_1}*i_2, \frac{\delta q_2^{(C)}}{i_1}*i_2, \frac{\delta q_3^{(C)}}{i_1}*i_2, \frac{\delta q_4^{(C)}}{i_1}*i_2, \frac{\delta q_5^{(C)}}{i_1}*i_2, \frac{\delta q_6^{(C)}}{i_1}*i_2\right)$$

This means that

$$\delta d_1^D = \frac{0.224}{7}*5 = 0.160, \quad \delta q_1^D = \frac{0.000786}{7}*5 = 0.000561$$

$$\delta a_1^D = \frac{0.173}{7}*5 = 0.124, \quad \delta q_2^D = \frac{0.000786}{7}*5 = 0.000561$$

$$\delta a_2^D = \frac{0.244}{7}*5 = 0.175, \quad \delta q_3^D = \frac{0.000786}{7}*5 = 0.000561$$

$$\delta a_3^D = \frac{0.178}{7} * 5 = 0.127\ ,\ \ \delta q_4^D = \frac{0.000628}{7} * 5 = 0.000448$$

$$\delta d_4^D = \frac{0.136}{7} * 5 = 0.097\ ,\ \ \delta q_5^D = \frac{0.000628}{7} * 5 = 0.000448$$

$$\delta d_6^D = \frac{0.150}{7} * 5 = 0.107\ ,\ \ \delta q_6^D = \frac{0.000628}{7} * 5 = 0.000448$$

We enter the data and check according to the forward direction of the software. According to the response from the calculation software, at the survey positions, 100% of the actual approach points of the end-effector are within the spherically permissible region. Thus, the problem meets the requirements.

The tolerances of the generalized coordinates and the link dimension tolerances for robot D were determined, where D is in the same group but does not have a similar structure as sample robot A.

VI.IV Adjusting tolerances in combination testing

The tolerance problem is solved independently for each parameter group (a_i, d_i) and (q_i). Combination testing of these values gives the position responses outside the sphere to describe the design quality. The adjustment of component tolerances is necessary by narrowing the larger tolerance domain or relative alignment between tolerance domains. The process is carried out on the software quickly. See the illustrated example below in which the values of the tolerances received are relatively equal.

Example: Calibrating tolerances between link parameters during checking.

We select the tolerances of the parameters of the links and joints from the results of the problem solved by the GRG method on Excel for the robot Fanuc s900w:

$$d_1^A = 335 \text{ mm} \quad \leftrightarrow \quad \delta d_1^A = \pm 0.167 \text{ (mm)}$$

$$a_1^A = 75 \text{ mm} \quad \leftrightarrow \quad \delta a_1^A = \pm 0.100 \text{ (mm)}$$

$$a_2^A = 270 \text{ mm} \quad \leftrightarrow \quad \delta a_2^A = \pm 0.160 \text{ (mm)}$$

$$a_3^A = 90 \text{ mm} \quad \leftrightarrow \quad \delta a_3^A = \pm 0.174 \text{ (mm)}$$

$$d_4^A = 295 \text{ mm} \quad \leftrightarrow \quad \delta d_4^A = \pm 0.100 \text{ (mm)}$$

$$d_6^A = 80 \text{ mm} \quad \leftrightarrow \quad \delta d_6^A = \pm 0.500 \text{ (mm)}$$

We place the values in a combined check with the resolution of the received variables as shown in Fig. 11. The result indicates that 510/91,800 points were tested outside the sphere.

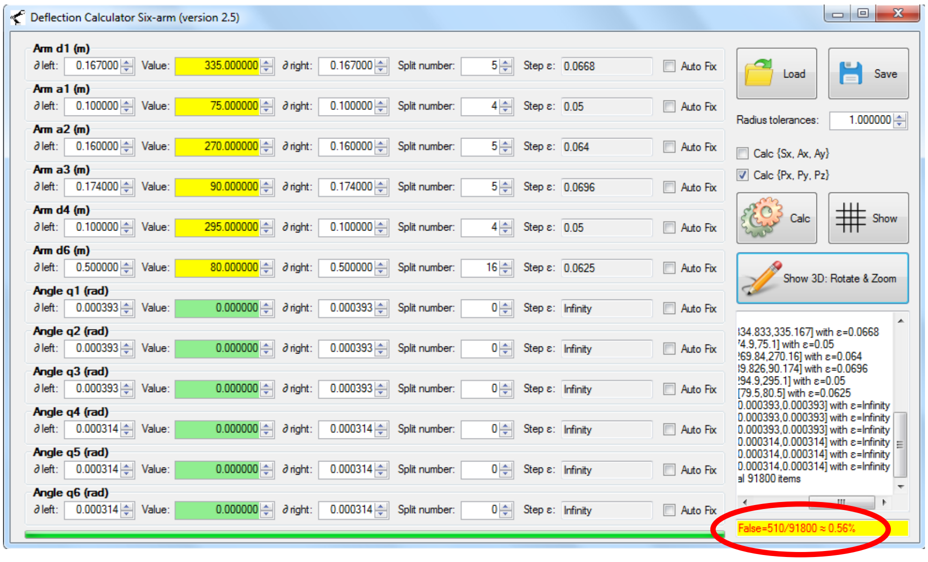


Fig. 11. Test results of end-effector accuracy at a survey position in software.

At a survey position in the workspace, with the selected tolerances, 0.56% of the points were outside the spherically permissible region (as shown in Fig. 12).

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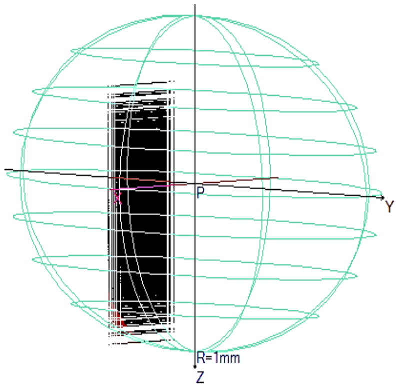

Fig. 12. Accuracy of the end-effector at a survey position.

We first calibrate tolerance by narrowing the larger tolerances to ensure a given reliability of the robot. Then, we enable the *auto-fix* function of the software as shown in Fig. 13. The tolerance domain is automatically reduced to 100% of the points within the sphere. Thus, the tolerance falls steadily to achieve the most economical production plan.

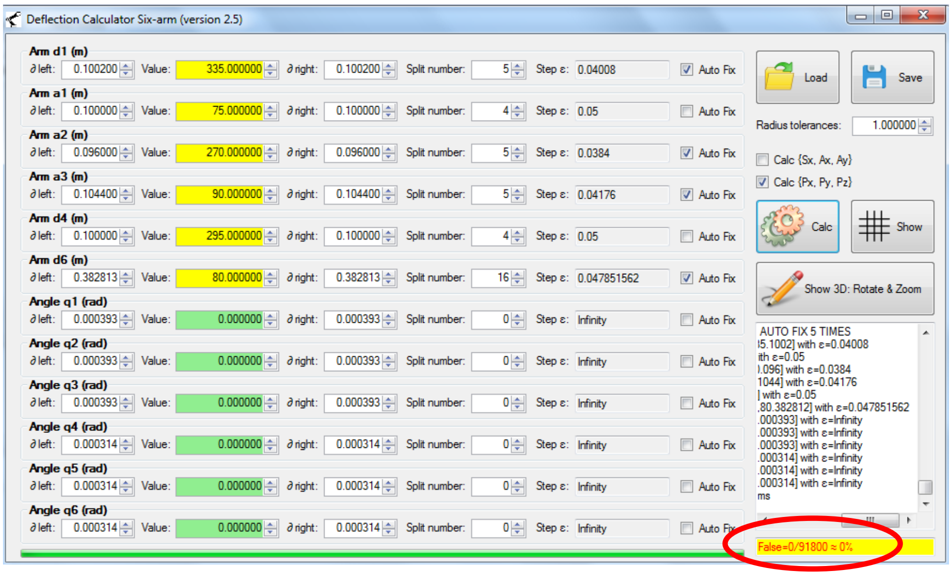

Fig. 13. Test results of end-effector accuracy at a survey position after auto-fix.

Specifically, the tolerance of the link parameter after calibration is given in Table 11.

Table 11: Kinematic parameter tolerances of robot A

δd_1^A (mm)	δa_1^A (mm)	δa_2^A (mm)	δa_3^A (mm)	δd_4^A (mm)	δd_6^A (mm)
±0.100	±0.100	±0.096	±0.104	±0.100	±0.382

Thus, it can be seen that based on solving inverse kinematic problems according to the GRG method, deviations of link lengths and joint angles are defined as the first approximations easily. The verification of the correctness of the results and the calibration of reasonable tolerances are performed by a computer based on the forward kinematic problem. With this approach, the kinematic parameter tolerances of a robot are quickly determined, saving considerable time for designers.

VII. CONCLUSION

This article developed a numerical method for calculating link length tolerances as well as generalized coordinate tolerances. In particular, the calculation results are still correct in cases where the links are interchangeably assembled in a large batch production. The division of the tolerance problem into two smaller and independent ones (the generalized coordinate tolerance and the link length tolerance) is reasonable. Testing of all possible combinations when interchangeably assembling two sets of these parameters was accomplished with a computer. We also maintained a reasonable adjustment of the link tolerances by an algorithm to ensure that there was no significant difference in value between them.

In addition, the hypothesis of the existence of the dimensional similarity ratio and accuracy ratio was confirmed. This factor is a basis for determining the tolerance of a group of robots when calculating the tolerance of a robot in that group. The use of this ratio as an intermediary for the logic inference when the robot has the same type of structure but is not similar to the sample robot was also pointed out. The simulation results in each case showed that the designed robots based on the relationship met the required precision.

The GRG method in accuracy design, used to solve the kinematic problem for the robot, has an advantage in that the GRG algorithm was used to solve the problem of separately identifying the link length tolerances and joint clearances applied in serial and parallel robots [17]. A tolerance calculation technique based on the similarity of the robot group reduces the computational time, especially when industrial robots have some typical structures. The software can be used for an entire group of robots because the parameters need to be changed only for specific situations.

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