

Optimization of Cost and CO₂ Emission in Reinforced Concrete Footings Using a Metaheuristic Algorithm: A parametric study

Carlos Millán-Páramo¹, Euriel Millán-Romero² and Fernando Jove Wilches¹

¹ Department of Civil Engineering, Universidad de Sucre, Sincelejo, Sucre, Colombia.

² Faculty of Engineering, Universidad de Sucre, Sincelejo, Colombia.

*Corresponding author: fernando.jove@unisucra.edu.co

ORCID: 0000-0002-0004-6063 (Carlos), 0000-0001-7955-9963 (Euriel), 0000-0002-2080-4036 (Fernando)

Abstract

It is necessary to build structural elements that are economical and that in turn support the loads to which they are subjected. On the other hand, it is important to control CO₂ emissions in construction processes. The excess of this gas is the main cause of global warming by the greenhouse effect. In this context, this work presents a sensitivity analysis to assess the impact of different design parameters on the optimization of cost and CO₂ emission of reinforced concrete footings. The design parameters are applied load, soil elastic modulus, Poisson ratio, angle of internal friction, factor of safety and allowable settlement. For this, the metaheuristic called Modified Simulated Annealing Algorithm is used to obtain the optimal designs.

Keywords: Reinforced concrete footings, cost and CO₂, optimal design, metaheuristics.

I. INTRODUCTION

The footings are structural members that serves as a foundation for a pillar, wall or other surface element, transmitting the efforts it receives from it to the ground. These can fail due to the shear break of the soil that supports it. However, prior to the occurrence of shear failure in the soil, and even if it does not, it is also possible that a superficial foundation is subjected to a settlement sufficiently large to cause damage to the structure and make it dysfunctional for the purpose for which was designed [1].

Designing economic structural elements is essential in engineering practice [2]. There are recently developed methods for the low-cost design of reinforced concrete footings [3,4] and this problem has been the subject of studies in the literature in the recent years. On the other hand, greenhouse gases have increased significantly due to human activity. Due to this, more sustainable design methods and construction practices are being used, which positively impacts the reduction of CO₂ emissions. However, reducing cost and CO₂ emissions is a difficult task to solve. This type of problems presents

multidimensional search spaces and a high number of restrictions, requiring a powerful optimization tool.

Modified simulated annealing algorithm (MSAA) is a simple single-solution algorithm based on the behavior of atomic arrangements in liquid or solid materials during the annealing process introduced by Millan et al. [5] and it has been shown to be a computationally efficient metaheuristic method to solve a variety of optimization problems [6–13]. MSAA is a newly improved version of the simulated annealing (SA) algorithm with three modifications. Firstly, a preliminary exploration is realized to choose the starting point of search. Secondly, the transition from the starting point to the new point is done by a search step. Thirdly, the range of probability of accepting a worse solution is reduced.

The aim of this work is performance a sensitivity analysis to assess the impact of different design parameters on the optimization of cost and CO₂ emission of reinforced concrete footings using Modified simulated annealing algorithm. The remainder of this paper is organized as follows. In Section 2, the methodology is described. Section 3 presents the results and discussions. Finally, in Section 4, our conclusions are presented.

II. METHODOLOGY

The procedure is divided into three stages: (i) formulation of geotechnical limit states (ii) formulation of the optimization design of reinforced spread footings and (ii) metaheuristic algorithm to solve the optimization problem.

II.I. Geotechnical limit states

Two criteria must be considered, regarding the geotechnical aspect in the design of isolated footings: achieving the base safety factor and not exceeding the permissible settlement value. Figure 1 shows the general dimensions of an isolated footing, assumed as squared. Geotechnical formulation is shown in Table 1.

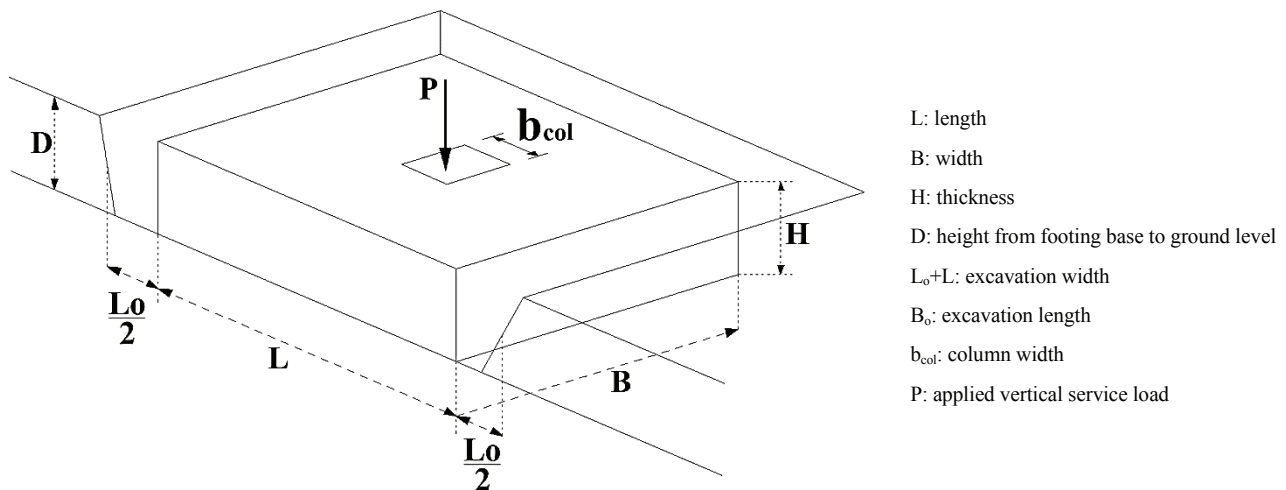


Fig. 1. Spread footing dimensions

Table 1. Geotechnical formulation

$q = \frac{P+W_f}{BL}$	(1)	$q \rightarrow$ uniformly distributed stress over the soil $W_f \rightarrow$ weight of any overload on the footing (including the weight of the footing itself)	
$FS_B = \frac{q_{ult}}{q}$	(2)	$FS_B \rightarrow$ base safety factor $q_{ult} \rightarrow$ is the ultimate bearing capacity of the footing	
$q_{ult} = cN_c F_{cs} F_{cd} + \gamma DN_q F_{qs} F_{qd} + 0.5\gamma BN_\gamma F_{\gamma s} F_{\gamma d}$	(3)	$\gamma \rightarrow$ foundation soil specific weight $c \rightarrow$ foundation soil cohesion $N_c, N_q,$ and $N_\gamma \rightarrow$ dimensionless bearing capacity factors (being only functions of the foundation soil friction angle - ϕ) $F_{cs}, F_{qs},$ and $F_{\gamma s} \rightarrow$ shape factors $F_{cd}, F_{qd},$ and $F_{\gamma d} \rightarrow$ depth factors	
$N_q = e^{\pi \tan \phi} \tan^2 \left(\frac{\pi}{4} + \frac{\phi}{2} \right)$	(4)	$\phi \rightarrow$ internal friction angle of the soil	
$N_\gamma = 2(N_q + 1) \tan \phi$	(5)		
$F_{qs} = 1 + \frac{B}{L} \tan \phi$	(6)		
$F_{\gamma s} = 1 - 0.4 \frac{B}{L}$	(7)		
$F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 U$	(8)		
$U = \left[\arctan \left(\frac{D}{B} \right) \right]$	(9)		
$F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \left(\frac{D}{B} \right)$	(10)		
$F_{\gamma d} = 1$	(11)		
$F_{cs} = 1 - \frac{BN_q}{LN_c}$	(12)		
F _{cd} values according to Arrieta [13]			
$\delta = \frac{(P+W_f)(1-\nu^2)}{\beta_z E \sqrt{BL}}$	(13)		$\delta \rightarrow$ settlements of the foundation $\nu \rightarrow$ Poisson's coefficient $E \rightarrow$ module of elasticity of the soil
$\beta_z = -0.0017 \left(\frac{L}{B} \right)^2 + 0.0597 \left(\frac{L}{B} \right) + 0.9843$	(14)		$\beta_z \rightarrow$ form factor

II.II. CO2 and cost optimization

The objective functions of cost and CO2 emissions for this problem are listed in Table 2.

Table 2. Formulation CO₂ and cost optimization

$f_{\text{cost}} = C_e V_e + C_f A_f + \xi C_r M_r + \frac{f_c}{f_{c \text{ min}}} C_c V_c + C_b V_b$ (15)	$C_e \rightarrow$ unit cost of excavation $C_f \rightarrow$ unit cost of the formwork $C_r \rightarrow$ unit cost of reinforcing steel $C_c \rightarrow$ unit cost of concrete $C_b \rightarrow$ unit cost of earth
$f_{\text{CO}_2} = E_e V_e + E_f A_f + \xi E_r M_r + \frac{f_c}{f_{c \text{ min}}} E_c V_c + E_b V_b$ (16)	$E_e \rightarrow$ unit emission of excavation $E_f \rightarrow$ unit emission of the formwork $E_r \rightarrow$ unit emission of the reinforcement $\xi \rightarrow$ factor scale that gives the reinforcement steel term a magnitude comparable to that of other terms $f_{c \text{ min}} \rightarrow$ minimum allowable strength of concrete
$V_e = (B+B_o)(L+L_o)D$ (17)	$\gamma \rightarrow$ foundation soil specific weight $c \rightarrow$ foundation soil cohesion $N_c, N_q,$ and $N_\gamma \rightarrow$ dimensionless bearing capacity factors (being only functions of the foundation soil friction angle - ϕ) $F_{cs}, F_{qs},$ and $F_{\gamma s} \rightarrow$ shape factors $F_{cd}, F_{qd},$ and $F_{\gamma d} \rightarrow$ depth factors
$A_f = 2H(B+L)$ (18)	
$M_r = mV_c$ (19)	$m \rightarrow 29.67 \text{ kg/m}^3$
$V_c = BLH - V_r$ (20)	
$V_b = [(B+B_o)(L+L_o) - BL]D$ for $H \geq B$ $V_b = V_e - [BLH + b_{\text{col}} l_{\text{col}}(D-H)]$ for $H < B$ (21)	
$f_{\text{multi}} = \zeta f_{\text{cost}} + (1-\zeta)f_{\text{CO}_2}$ (22)	

II.III. Modified simulated annealing algorithm (MSAA)

The MSAA is a single-solution metaheuristic based on the cooling of metals phenomenon and it has three main stages that

differentiate it from the simulated annealing (SA). Table 3 summarizes these characteristics. For more details, see [5].

Table 3. Modified simulated annealing algorithm

The starting point is selected by a preliminary exploration	$X_{P \times N} = I_{P \times N} X_L + \text{rand}_{P \times N}(X_U - X_L)$ (23)	$P \rightarrow$ number of points (states) that are desired in the search space $N \rightarrow$ number of dimensions of the problem $I_{P \times N} \rightarrow$ identity matrix of size $P \times N$ $X_L \rightarrow$ lower limit of the problem $X_U \rightarrow$ upper limit of the problem $\text{rand}_{P \times N} \rightarrow$ matrix of random numbers (pure randomness) between 0 and 1 of size $P \times N$.
The transition from the starting point to the new point is performed by the addition of random numbers that are within the defined radius.	$R_{i+1} = R_i \cdot \alpha$ (24)	$R_i \rightarrow$ initial radius $\alpha \rightarrow$ radius reduction coefficient
The probability of accepting a worse solution is reduced.	$P = \frac{1}{1 + e^{(\Delta f/T)}}$ (25)	$P \rightarrow$ probability of accepting the new state $\Delta f \rightarrow$ difference of the evaluations of the function for each state $T \rightarrow$ temperature of the system $e \rightarrow$ Euler number

The design parameters used for this problem and the value limits for the design variables are listed in Table 4 [2]. The exercise is restricted by a safety factor of 3.0 and maximum settlements of 25 mm.

Table 4. Input parameters and Design variables

Input parameter	Symbol	Value
Internal friction angle of soil (°)	ϕ	35
Unit weight of soil (kN/m ³)	γ_s	18.5
Poisson ratio of soil	ν	0.3
Modulus of elasticity of soil (MPa)	E	50
Applied vertical force (kN)	P	3000
Over excavation length (m)	L_o	0.3
Over excavation width (m)	B_o	0.3
Thickness of footing (m)	H	0.6
Factor of safety for bearing capacity	FS	3
Maximum allowable settlement (mm)	δ	25
Design variables		
	Lower bound	Upper bound
B (m)	0.01	5.0
L (m)	0.01	5.0
D (m)	0.50	2.0

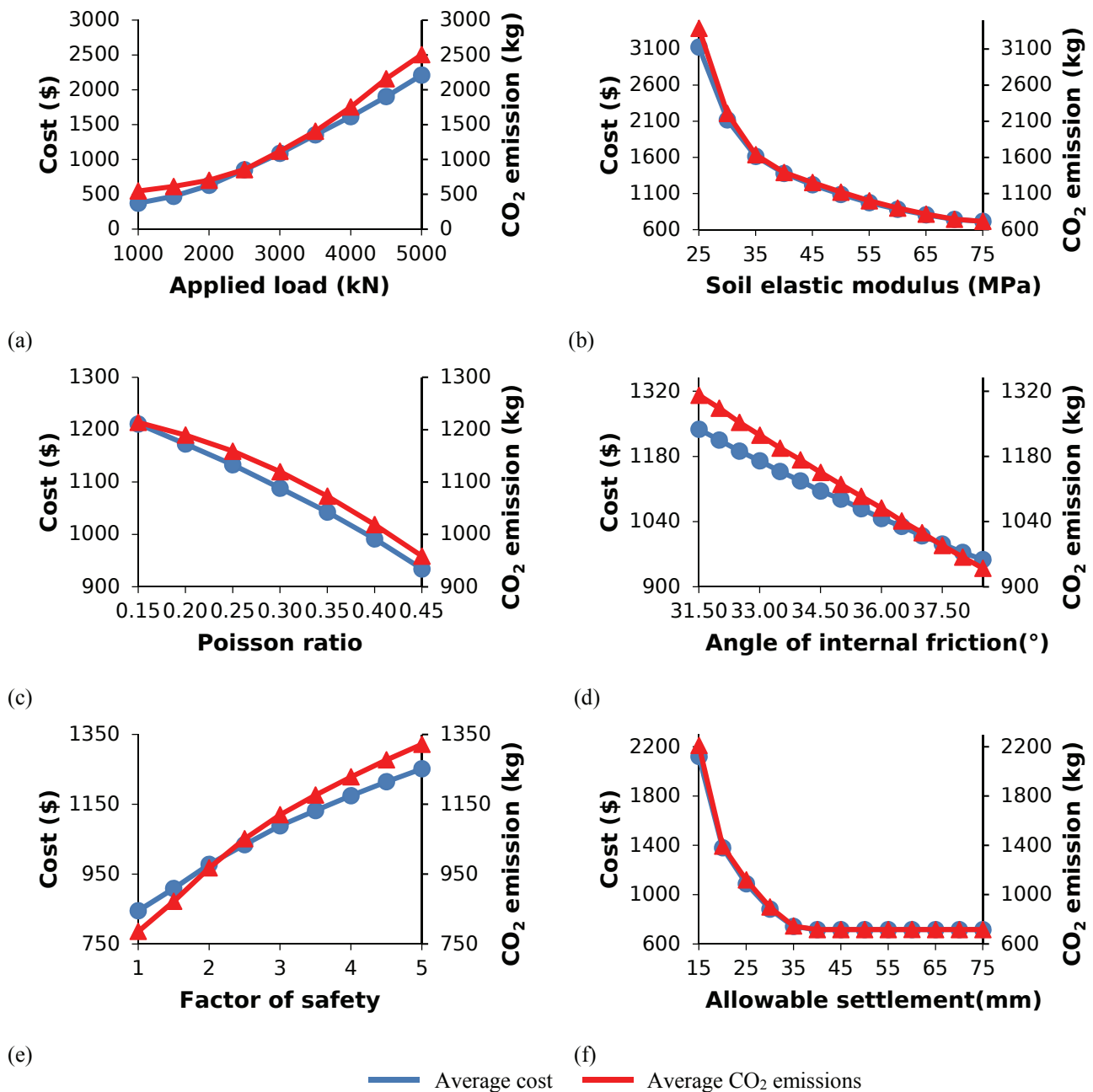


Figure 2. Effects of: (a) applied load, (b) soil elastic modulus, (c) Poisson ratio, (d) angle of internal friction, (e) factor of safety and (f) allowable settlement on average cost and average CO₂ emission designs

III. RESULTS AND DISCUSSIONS

100 runs were executed, and the average cost and CO₂ emission designs are calculated for a typical range of applied load, soil elastic modulus, Poisson ratio, angle of internal friction, factor of safety and allowable settlement. While the number of runs is arbitrary, it should be adequate to provide reliable statistics on the general quality of the solutions and the convergence of the MSAA. It is also important to note that all presented MSAA designs are feasible. The algorithm was coded in Matlab.

Figure 2 shows the effects of design parameters on cost and CO₂ emission in reinforced concrete footings. Figure 2(a) shows that the cost and CO₂ emissions increase as the applied column load increases. Figure 2(b) shows that as the soil becomes more rigid, the cost and CO₂ emission values decrease considerably. Figure 2(c) shows that as the soil Poisson's ratio increases, the cost and CO₂ emission values decrease slightly. Figure 2(d) shows that as the internal friction angle of the soil increases, the cost and CO₂ emission values decrease. Figure 2(e) shows that as the required minimum safety factor increases, the cost and CO₂ emission values increase. Figure 2(f) shows that as the maximum allowed settlement increases, the cost and CO₂ emissions of the footing designs are significantly reduced to a point where the settlement no longer controls the project ($\delta > 35$ mm).

VI. CONCLUSIONS

In this paper the impact of different design parameters on the optimization of cost and CO₂ emission in reinforced concrete footings using a metaheuristic algorithm was assessed. The metaheuristic algorithm implemented was Modified Simulated Annealing Algorithm. From the results obtained, it is concluded that both cost and CO₂ emissions are highly sensitive to changes in the applied load, in the modulus of soil elasticity and in the maximum allowed settlement. The soil Poisson's ratio, the internal friction angle and the safety factor have less impact on the cost and CO₂ emission values.

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