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A Generalised Fixed Point Theorem for Set Valued Presic Type Contractions in a Metric Space

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ABSTRACT

A generalised common fixed point theorem for Presic Type Hybrid Contraction involving two maps $f : X \to X$ and $T : X^k \to CB(X)$ in metric space is proved. The result generalises many known results.

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1. INTRODUCTION

Nadler[11] introuced set valued contractive mappings in metric spaces and proved existence of fixed points for such mappings. Later many authors extended and generalised the work of Nadler in different directions.

Considering the convergence of certain sequences, Presic [13] proved the following :

Theorem 1.1. *Let (X, d) be a metric space, k a positive integer, $T : X^k \longrightarrow X$ be a mapping satisfying the following condition :*

$$d(T(x_1, x_2, \dots, x_k), T(x_2, x_3, \dots, x_{k+1})) \leq q_1.d(x_1, x_2) + q_2.d(x_2, x_3) + \cdots + q_k.d(x_k, x_{k+1}) \tag{1.1}$$

where $x_1, x_2, \dots, x_{k+1}$ are arbitrary elements in X and $q_1, q_2, \dots, q_k$ are non-negative constants such that $q_1 + q_2 + \cdots + q_k < 1$. Then, there exists some $x \in X$ such that $x = T(x, x, \dots, x)$. Moreover if $x_1, x_2, \dots, x_k$ are arbitrary points in X and for $n \in N$ $x_{n+k} = T(x_n, x_{n+1}, \dots, x_{n+k-1})$, then the sequence $< x_n >$ is convergent and $lim\, x_n = T(lim\, x_n, lim\, x_n, \dots, lim\, x_n)$.

Note that for $k = 1$, the above theorem reduces to the well-known Banach Contraction Principle.
Ciric and Presic [5] generalising the above theorem proved the following:

Theorem 1.2. *Let (X, d) be a metric space, k a positive integer, $T : X^k \longrightarrow X$ be a mapping satisfying the following condition :*

$$d(T(x_1, x_2, \dots, x_k), T(x_2, x_3, \dots, x_{k+1})) \leq \lambda.max\{d(x_1, x_2), d(x_2, x_3), \dots d(x_k, x_{k+1})\} \tag{1.2}$$

where $x_1, x_2, \dots, x_{k+1}$ are arbitrary elements in X and $\lambda \in (0, 1)$. Then, there exists some $x \in X$ such that $x = T(x, x, \dots, x)$. Moreover if $x_1, x_2, \dots, x_k$ are arbitrary points in X and for $n \in N x_{n+k} = T(x_n, x_{n+1}, \dots, x_{n+k-1})$, then the sequence $< x_n >$ is convergent and $lim\, x_n = T(lim\, x_n, lim\, x_n, \dots, lim\, x_n)$. If in addition T satisfies $D(T(u, u, \dots u), T(v, v, \dots v)) < d(u, v)$, for all $u, v \in X$ then x is the unique point satisfying $x = T(x, x, \dots, x)$.

The above theorems have been generalised by Pacurar [12], Reny George et al [9]. Nadler [11] generalised the Banach Contraction to set valued functions and proved the following:

Theorem 1.3. *Let (X, d) be a complete metric space and T be a mapping from X into $CB(X)$ where $CB(X)$ denotes all closed bounded subsets of X such that for all x, $y \in X$,*

$$H(Tx, Ty) \leq \lambda d(x, y) \tag{1.3}$$

where $0 \leq \lambda < 1$. Then T has a fixed point.

Several fixed point theorems have been established in various topological spaces using Set Valued / hybrid contractions. (see [1-4],[6-8],10,15,17). The present paper is aimed at proving fixed point theorems for set valued mappings of Presic Type there by generalising the above theorems and other proven results.

2. PRELIMINARIES

The following definitions are needed in the sequel:

Definition 2.1. *Let (X, d) be a metric space, k a positive integer, $T : X^k \to X$ and $f : X \to X$ be mappings.*

(a) An element $x \in X$ is said to be a *coincidence point* of f and T if and only if $f(x) \in T(x, x, ..., x)$. If $x \in f(x) = T(x, x, ..., x)$ then we say that x is a $common\ fixed\ point$ of f and T.

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(b) Mappings f and T are said to be *commuting* if and only if $f(T(x,x,...x)) = T(fx, fx, ...fx)$ for all $x \in X$.

(c) Mappings f and T are said to be *weakly compatible* if $fx \in T(x,x,......x)$ implies $T(f(x,x,...) \subseteq T(fx, fx...fx)$.

Remark 2.2. *for $k = 1$, the above definitions reduces to the usual definition of commuting and weakly compatible mappings in a metric space.*

The set of coincidence points of f and T is denoted by $C(f,T)$.

Definition 2.3. *Let A be a non-empty subset of a metric space (X, d). For $x \in X$, define*

$$d(X,A) = inf\{d(x,y) : y \in A\} \tag{2.1}$$

Let $CB(X)$ denote the set of all nonempty closed bounded subsets of X. For A, B $\in CB(X)$, define

$$\delta(A,B) = sup\{d(x,B) : x \in A\} \tag{2.2}$$

$$H(A,B) = max\{\delta(A,B), \delta(B,A)\} \tag{2.3}$$

Then H is a metric on $CB(X)$ and is called Hausdorff metric.

Remark 2.4. *Let (X,d) be a metric space and A,B $\in CB(X)$. Then for all $\epsilon > 0$ and $a \in A$, there exists a point $b \in B$ such that $d(a,b) \le H(A,B) + \epsilon$.*

3. MAIN RESULTS

Consider a function $\phi : R^k \to$ R such that
(a) ϕ is an increasing function,*i.e* $x_1 \le y_1, x_2 \le y_2, ...x_k \le y_k$ implies $\phi(x_1, x_2, ...x_k) \le \phi(y_1, y_2, ...y_k)$.
(b) $\phi(t,t,t,...) \le t$, for all $t \in X$
(c) ϕ is continuous in all variables (d) $\phi(t_1, t_2, ...t_k \le$ Max $(t_1, t_2, ...t_k)$

Now we present our main results as follows :

Theorem 3.1. *Let (X,d) be a metric space. For any positive integer k, let $T : X^k \to CB(X)$ and $f : X \to X$ be mappings satsifying,*

$$T(X^K) \subseteq f(X) \tag{3.1}$$

$$\begin{aligned} &H(T(x_1, x_2, ...x_k), T(x_2, x_3, ...x_{k+1})) \\ &\le \lambda\phi(d(fx_1, fx_2), d(fx_2, fx_3), ...(fx_k, fx_{k+1})) \end{aligned} \tag{3.2}$$

where $x_1, x_2, ...x_{k+1}$are arbitrary elements in X and $\lambda \in (0,1)$ and

$$\text{f(X) is complete} \tag{3.3}$$

then f and T have a coincidence point, i.e., i.e. $C(f,T) \neq \emptyset$

Proof. Let $R > 0$ where,

$$\begin{aligned} R = max\{&\frac{d(fx_1, fx_2)}{\theta}, \frac{d(fx_1, fx_2)}{\theta^2}, \\ &\frac{d(fx_1, fx_2)}{\theta^3}, ...\frac{d(fx_1, fx_2)}{\theta^k}\} \end{aligned} \tag{3.4}$$

with $\theta = \lambda^{\frac{1}{k}}$ $\{\epsilon\} \subset X$ be a sequence satisfying

$$0 < \epsilon_n \text{ and } \epsilon_i \le R\theta^{k+i} \tag{3.5}$$

Let $x_1,\ x_2,\ ....x_n$ be arbitary elements in X. By (3.1), (3.4) and remark (2.4), there exist $x_{k+1},\ x_{k+2}$ in X, such that $y_{k+1} = fx_{k+1} \in T(x_1, x_2.....x_k)$ and $y_{k+2} = fx_{k+2} \in T(x_2, x_3.....x_{k+1})$ such that

$d(y_{k+1}, y_{k+2}) = d(fx_{k+1}, fx_{k+2})$
$\le H(T(x_1, x_2, ...x_k), T(x_2, x_3, ...x_{k+1}) + \epsilon_1$
$\le \lambda\phi(d(fx_1, fx_2), d(fx_2, fx_3), ...(fx_k, fx_{k+1})) + \epsilon_1$
$\le \lambda\phi(R\theta, R\theta^2, ...R\theta^k) + \epsilon_1$
$\le \lambda\phi(R\theta, R\theta, ...R\theta) + \epsilon_1$
$\le \lambda(R\theta) + \epsilon_1 \le R\theta^{k+1} + \epsilon_1$
$\le 2R\theta^{k+1}$

Also,

$d(y_{k+2}, y_{k+3}) = d(fx_{k+2}, fx_{k+3})$
$\le H(T(x_2, x_3, ...{}_{k+1}), T(x_3, x_4, ...x_{k+2}) + \epsilon_2$
$\le \lambda\phi(d(fx_2, fx_3), d(fx_3, fx_4), ...(fx_{k+1}, fx_{k+2})) + \epsilon_2$
$\le \lambda\phi(R\theta^2, R\theta^3, ...2R\theta^{k+1}) + \epsilon_2$
$\le \lambda\phi(R\theta^2, R\theta^2, ...R\theta^2) + \epsilon_2$
$\le \lambda R\theta^2) + \epsilon_2 \le \lambda 2R\theta^{k+1} + \epsilon_2$
$\le 3R\theta^{k+2}$

Continuing this process we can form the sequence $< y_n >$ such that y_{n+k}=$fx_{n+k} \in T(x_n, x_{n+1}, x_{n+2}....x_{n+k-1})$ with

$$d(y_n, y_{n+1}) \le (n+1)R\theta^n \tag{3.6}$$

for all n. Now, for p, $n \in N$, we have
$d(y_n, y_{n+p}) \le d(y_n, y_{n+1}), d(y_{n+1},$
$y_{n+2}), ...d(y_{n+p-1}, y_{n+p})$
$\le (n+1)R\theta^n + (n+2)R\theta^{n+1} + ...(n+p)R\theta^{n+p-1}$
$= nR\theta^n \sum_{i=0}^{p-1} \theta^i + R\theta^n \sum_{i=1}^{p} i\theta^{i-1}$
As, $n \to \infty$, we have $d(y_n, y_{n+p}) \to 0$
Clearly $\{y_n\}$ is Cauchy sequence and since $f(X)$ is complete, we have some u, $v \in X$ such that $lim_{n\to\infty} y_n$ = v = fu. Now,

$d(fu, T(u,u,...u)) \le d(fu, fx_{n+k})$
$+d(fx_{n+k}, T(u,u,...u))$
=$d(fu, fx_{n+k})$+ $H(T(x_n, x_{n+1}, ...x_{n+k-1}), T(u,u,....u))$
$\le$ $d(fu, fx_{n+k})$ + $H(T(u,u,....u), T(u,u,...x_n))$ + $H(T(u,u,....x_n), T(u,u,...x_n,x_{n+1}))$+.....
+........$H(T(u, x_n, x_{n+1}, ...x_{n+k-2}), T(x_n, x_{n+1}, ...x_{n+k-1}))$

The above reduces to,

$d(fu, T(u,u,...u)) \le d(fu, y_{n+k})$
$+\lambda\phi\{d(fu,fu), d(fu,fu), \ldots, d(fu, fx_n)\}$
$+\lambda\phi\{d(fu,fu), d(fu,fu), \ldots, d(fu, fx_n), d(fx_n, fx_{n+1})\}+ \cdots$
$+\lambda\phi\{d(fu, fx_n), d(fx_n, fx_{n+1}), \ldots d(fx_{n+k-2}, fx_{n+k-1})\}$.
$= d(fu, y_{n+k}) + \lambda\phi(0,0,\ldots, d(fu, fx_n))$
$+\lambda\phi(0,0,\ldots, d(fu, fx_n), d(fx_n, fx_{n+1})) + \cdots$
$+\lambda\phi(d(fu, fx_n), d(fx_n, fx_{n+1}), \ldots d(fx_{n+k-2}, fx_{n+k-1}))$.
$= d(fu, y_{n+k}) + \lambda\phi(d(fu, fx_n),$

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$d(fx_n, fx_{n+1}), \dots d(fx_{n+k-2}, fx_{n+k-1}))$.

Taking, $lim_{n\to\infty}$ in RHS, we get,
$d(fu, T(u, u, ...u)) = 0$.
As, $T(u, u, ...u) \in CB(X)$, we have, $v = fu \in T(u, u, u, ...u)$
i.e., u is coincidence point and v is point of coincidence of f and T.
Thus $C(f, T) \neq \emptyset$

□

Theorem 3.2. *Let (X, d) be a metric space. For any positive integer k, let $T : X^k \to X$ and $f : X \to X$ be two mappings satisfying all conditions of Theorem (3.1), with*

$$u \in C(f, T) \Rightarrow T(u, u, u, ...u) = \{fu\} \quad (3.7)$$

with $\lambda \in (0, \frac{1}{k})$. Then T and f have unique point of coincidence. Further if f and T are weakly compatible, then f and T have a unique common fixed point.

Proof. From Theorem 3.1, $C(f, T) \neq \emptyset$ with $u \in C(f, T)$. i.e. $f(u) = T(u, u, u.....u) = v$ for some $v \in X$. So v is a point of coincidence of f and T. We claim the point of coincidence is unique. If not, let there exist u' in X such that
$\{v\} = \{fu\} = T(u, u, ...u)$ and
$\{v'\} = \{fu'\} = T(u', u', ...u')$
$d(v', v) = H(\{v'\}, \{v\})$
$d(v', v) = H(T(u', u', u', ...u'), T(u, u, u, ...u))$
$\leq H(T(fu', u', ...u'), T(u', u', u', ...u)) +$
$H(T(u', u', ...fu, u), T(u', u', u', ...u, u))+$
$.......H(T(u', u, u, ...u, u), T(u, u, ...u))$
$\leq \lambda\phi\{d(fu', fu'), d(fu', fu'), ..., d(fu', fu)\}$
$+\lambda\phi\{d(fu', fu'), d(fu', fu'), ...d(fu', fu), d(fu, fu)\} +$
...
$+\lambda\phi\{d(fu', fu), d(fu, fu), ...d(fu, ffu)\}$.
$= \lambda\phi(0, 0, ..., d(fu', fu))$
$+\lambda\phi(0, 0, ...d(fu', fu), 0) + ...$
$+\lambda\phi(d(fu', fu), 0, 0, 0, ...0))$.
$= k\lambda d(fu', fu)$.
$= k\lambda d(v', v)$
Repeating n times, we get, $d(v', v) \leq k^n\lambda^n d(v', v)$
As $n \to \infty$, $k^n\lambda^n \to 0$ and hence $d(v', v) \to 0$.
So $v = v'$, i.e. point of coincidence of f and T is unique. Since f and T are weakly compatible, we have,
$fv = ffu = f(T(u, u, ...u)) \in T(fu, fu, ...fu)$
i.e., $fv \in T(v, v, ...v) = w$
Since Point of Coincidence is unique, $w = v$.
Therefore, $fv = T(v, v, v, ...v) = v$
So v is unique common fixed point of f and T. □

Remark 3.3. *For k = 1 and f = Id (identity mapping), Theorem 3.2 becomes set valued contraction of Nadler type.*

Example 3.4. *Let $X = R^2, X = [0, 2]$ and $d : X \times X \to R$ such that $d(x, y) =| x - y |$. Then d is a metric on X Let A be the collection of all non empty subsets of X of the form $A = \{[0, x] : x \in X\}$. Denote $H : A \times A \to X$ with respect to d as follows: $H(A, B) =| x - y |$ for $A = [0, x]$ and $B = [0, y]$. $T : X^2 \to X$ and $f : X \to X$be defined as follows.*
$T(x, y)$=*[*$\frac{0,(x^2+y^2)}{4}+\frac{1}{2}$*] if* $(x, y) \in [0, 1] \times [0, 1]$
$T(x, y)$=*[*$\frac{0,(x+y)}{4}+\frac{1}{2}$*]if* $(x, y) \in [1, 2] \times [1, 2]$
$T(x, y)$=*[*$\frac{0,(x^2+y)}{4}+\frac{1}{2}$ *]if* $(x, y) \in [0, 1] \times [1, 2]$
$T(x, y)$=*[*$\frac{0,(x+y^2)}{4}+\frac{1}{2}$*] if* $(x, y) \in [1, 2] \times [0, 1]$
$f(x) = x^2$ *if* $x \in [0, 1]$
$f(x)$=x *if* $x \in [1, 2]$

Case 1. $x, y, z \in [0, 1]$
$H(T(x, y), T(y, z))$= $| T(x, y) - T(y, z) |$
=$| \frac{(x^2+y^2)}{4} + \frac{(y^2+z^2)}{4} |$
=$| \frac{(x^2-y^2)}{4} + \frac{(y^2-z^2)}{4} |$
$\leq \frac{1}{2}.max\{d(fx, fy), d(fy, fz)\}$

Case 2. $x, y \in [0, 1]$ *and* $z \in [1, 2]$
$H(T(x, y), T(y, z))$= $| \frac{(x^2+y^2)}{4} - \frac{(y^2+z)}{4} |$=$| \frac{(x^2-y^2)}{4} + \frac{(y^2-z)}{4} |\leq \frac{1}{2}.max\{d(fx, fy), d(fy, fz)\}$

Case 3. $x \in [0, 1]$ *and* $y, z \in [1, 2]$
$H(T(x, y), T(y, z))$= $| \frac{(x^2+y)}{4} - \frac{(y+z)}{4} |$=$| \frac{(x^2-y)}{4} + \frac{(y-z)}{4} |\leq \frac{1}{2}.max\{(fx, fy), d(fy, fz)\}$

Case 4. $x, y, z \in [1, 2]$
$H(T(x, y), T(y, z))$= $| \frac{(x+y)}{4} - \frac{(y+z)}{4} |$=$| \frac{(x-y)}{4} + \frac{(y-z)}{4} |\leq \frac{1}{2}.max\{(fx, fy), d(fy, fz)\}$
Similarly in all other cases we see that $d(T(x, y), T(y, z)) \leq \frac{1}{2}.max\{(fx, fy), d(fy, fz)\}$
and f and T satisfy the condition (3.2) with $\phi(x_1, x_2) = max(x_1, x_2)$. It can be seen that $C(T, f) = \{0, 1\}$, f and T commute at 0 and 1 and so are weakly compatible. Finally, $Fix(f, T) = \{0, 1\}$. f and T do not satisfy condition (3.6). Hence the Fixed point is not unique.

Example 3.5. *Let $X = R^2, X = [0, 2]$ and $d : X \times X \to R$ such that $d(x, y) =| x - y |$. Then d is a metric on X. Let A be the collection of all non empty subsets of X of the form $A = \{[0, x] : x \in X\} \bigcup \{\{x\} : x \in X\}$. Denote $H : A \times A \to X$ with respect to d as follows:*

$$H(A, B) = \begin{cases} | x - y | \; for A = [0, x] and B = [0, y]. \\ | x - y | \; for A = \{x\} and B = \{y\}. \\ max\{y, | x - y |\} for A = [0, x] and B = \{y\} \\ max\{x, | x - y |\} for A = \{x\} and B = [0, y] \end{cases}$$

$T : X^2 \to X$ and $f : X \to X$be defined as follows.
$T(x, y)$=*[*$\frac{0,(x^2+y^2)}{8}$*] if* $(x, y) \in [0, 1] \times [0, 1]$
$T(x, y)$=*[*$\frac{0,(x+y)}{8}$*]if* $(x, y) \in [1, 2] \times [1, 2]$
$T(x, y)$=*[*$\frac{0,(x^2+y)}{8}$*]if* $(x, y) \in [0, 1] \times [1, 2]$

$T(x,y)$=*[* $\frac{0,(x+y^2)}{8}$ *] if* $(x,y) \in [1,2] \times [0,1]$

$$f(x) = \begin{cases} \frac{x^2}{2} if x \in [0,1] \\ x if x \in [1,2] \end{cases}$$

As in previous example we can show that f *and* T *satisfy all the conditions of Theorem (3.1) with* $\phi(x_1, x_2) = max(x_1, x_2)$ *and* $\frac{\lambda=1}{4}$. *Clearly* $C(f,t) = \{0\}$ *and* $T(0,0) = 0$. *Thus all the conditions of Theorem 3.2 are satisfied and* $Fix(f,t) = \{0\}$.

Corollary 3.6. *Let* (X,d) *be a metric space. For any positive integer* k, *let* $T : X^k \to CB(X)$ *and* $f : X \to X$ *be mappings such that* $f(X)$ *is a closed subspace of* X *and* $T(x_1, x_2, ..., x_k) \subset f(X)$ *for all* x_1, x_2,....x_k, *satisfying the following conditions:*

$$f(X) \text{ is complete} \tag{3.8}$$

$$H(T(x_1, x_2, ...x_k), T(x_2, x_3, ...x_{k+1})) \leq \sum_{i=1}^{k} \alpha_i d(fx_i, fx_{i+1}) \tag{3.9}$$

where $x_1, x_2, ...x_{k+1}$*are arbitrary elements in* X *and* α_i *are non negative constants such that* $\sum_{i=1}^{k} \alpha_i < 1$. *Then* f *and* T *have a point of coincidence,* $v \in X$. *and Further if* f *and* T *are weakly compatible, then* f *and* T *have a common fixed point.*

Proof. Proof follows from Theorem 3.1 and 3.2 by taking $\phi(t_1, t_2....t_k) \leq Max((t_1, t_2....t_k)$. □

Taking f = Ix in the above Corollary and proceeding as before, we get the following :

Corollary 3.7. *Let* (X,d) *be any complete metric space,* k *a positive integer. Let* $T : X^k \to CB(X)$ *be a set valued Presic Type contraction, with* $\sum_{i=1}^{k} \alpha_i < 1$, *Then* T *has a fixed point* $v \in X$.

Remark 3.8. *Cor (3.6) and (3.7) are the main theorems 3 and 4 of Sukhla et al [16]. Corollary (3.6) is an extension of the theorem of Nadler [11] in product spaces which also generalises Theorem 1 for set valued mappings.*

Let us consider the following:
Let $\phi : R^k \to$ R such that
(a) ϕ is an increasing function,$i.e$ $x_1 \ll y_1, x_2 \ll y_2, ...x_k \ll y_k$ implies $\phi(x_1, x_2, ...x_k) \ll \phi(y_1, y_2, ...y_k)$.
(b) $\phi(t, t, t, ...) \leq t$, for all $t \in X$
(c) ϕ is continuous in all variables.

Theorem 3.9. *Let* (X,d) *be any metric space,* λ *a positive integer, Let* $T : X^k \to CB(X)$ *and* $f : X \to X$ *be two mappings such that* $T(x_1, x_2, ..., x_k) \subset f(X)$ *for all* x_1, x_2,....x_k, *satisfying the following conditions:*

$$\begin{aligned} &H(T(x_1, x_2, ...x_k), T(x_2, x_3, ...x_{k+1})) \\ &\leq \lambda\phi(d(fx_i, fx_{i+1})), for i = 1 to k \end{aligned} \tag{3.10}$$

where $x_1, x_2, ...x_{k+1}$*are arbitrary elements in* X. *Suppose there exists* $u \in X$ *such that*

$$d(fu, T(u, u, u, u...u) \leq d(fx, T(x, x, ...x) for all x \in X \tag{3.11}$$

Then f *and* T *have a point of coincidence,* $v \in X$ *and Further if* f *and* T *are weakly compatible, then* f *and* T *have a common fixed point.*

Proof. Let $G(x) = d(fx, T(x, x, ...x))$, for all $x \in X$. Then we have,

$$G(u) \leq G(x) for all x \in X \tag{3.12}$$

If $v = fu \in T(u, u, ...)$, then u is a coincidence point and v is point of coincidence of f and T. If not,
$G(u) = d(fu, T(u, u, ...u)) > 0$. Since $T(u, u, ...u) \in CB(X)$ and $T(u, u...u) \subset f(X)$, let $y = fz = T(u, u, ...u)$ be aribitary.
From (3.11) we have, $G(u) \leq G(z) = d(fz, T(z, z, ...z)$
$\leq H(T(u, u, ...u), T(z, z, ...z)$
$\leq H(T(u, u, ...u), T(u, u...u, z))$
$+H(T(u, u, ..., u, z), T(u, u...u, z, z))$
$+H(T(u, z, ..., z, z), T(z, z...z, z))$
$\leq \lambda\phi d(fu, fz) + \lambda\phi d(fu, fz) + ...\lambda\phi d(fu, fz)$
$\leq \lambda d(fu, fz)$
$\Rightarrow G(u) \leq \lambda d(fu, y)$, for all $y \in T(u, u, ...u)$
$\Rightarrow G(u) \leq \lambda d(fu, T(u, u, ...u))$
Since $0 < \lambda < 1$, we have $G(u) < \lambda G(u)$, which is a contradiction.
$\Rightarrow G(u) = 0$, i.e., $d(fu, T(u, u, ...u) = 0$.
$\Rightarrow v = fu \in T(u, u, ...u)$
Thus u is a coincidence point and v is point of coincidence of f and T. i.e., $C(f, T) \neq \emptyset$.
The rest of the proof follows from Theorem 3.2. □

Corollary 3.10. *Let* (X,d) *any metric space. For any positive integer* k, *let* $T : X^k \to CB(X)$ *and* $f : X \to X$ *be mappings such that* $f(X)$ *is a closed subspace of* X *and* $T(x_1, x_2, ..., x_k) \subset f(X)$ *for all* x_1, x_2,....x_k, *satisfying the following conditions:*

$$H(T(x_1, x_2, ...x_k), T(x_2, x_3, ...x_{k+1})) \leq \sum_{i=1}^{k} \alpha_i d(fx_i, fx_{i+1}) \tag{3.13}$$

where $x_1, x_2, ...x_{k+1}$*are arbitrary elements in* X *and* α_i *are non negative constants such that* $\sum_{i=1}^{k} \alpha_i < 1$. *Suppose there exists* $u \in X$ *such that*

$$d(fu, T(u, u, ...u) \leq d(fx, T(x, x, ...x)) for all x \in X, \tag{3.14}$$

then f *and* T *have a point of coincidence,* $v \in X$. *Further if* f *and* T *are weakly compatible, then* f *and* T *have a common fixed point.*

Proof. Proof follows from Theorems (3.9) and (3.2). □

Remark 3.11. *The above corollary corresponds to Theorem 5 and 6 of Sukhla et al [16].*

Corollary 3.12. *Let* (X,d) *be any metric space,* k *a positive integer. Let* $T : X^k \to CB(X)$ *be a set valued Presic Type contraction, with* $\sum_{i=1}^{k} \alpha_i < 1$. *Suppose there exists* $u \in X$ *such that*

$$d(u, T(u, u, ...u) \leq d(x, T(x, x, ...x)) \; for \; all \; x \in X, \tag{3.15}$$

Then T has a fixed point $v \in X$.

Proof. Taking $f = Ix$ in the above Corollary and proceeding as before, the result will follow. □

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