

# Spatial Distribution for Enterprises of Innovative Clusters in Economic Space of Region

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## Abstract

The subject of the study is the improvement of methodological tools for solving the problems of spatial localization of innovative clusters in the region. The aim of the work is to develop mathematical model of clustering economy, expanding the methodological potential of optimal placement of cluster initiatives in the economic space. Special attention is paid to the peculiarities of the innovative cluster development process, which determine the initial conditions of the problem of finding the optimal spatial location and long-term development of industrial production in the cluster. The problems of optimization of spatial distribution of new and development of existing industries in the process of implementation of the cluster initiative are considered, approaches to mathematical modeling of formation and development of innovative clusters are generalized. The specificity for solving the problems of optimizing the spatial distribution of enterprises of regional industrial clusters is presented. In General, the problem of optimal spatial distribution and long-term development of production in the cluster is formulated. Criteria of optimality of spatial placement and perspective development of the enterprises of innovative cluster are developed.

**Keywords:** innovative clusters, spatial development, economic and mathematical modeling, regional economy, localization of production, optimization problems, clustering of the economy.

## 1. INTRODUCTION

The optimal structure of location and coordinated development of enterprises in clusters is important for scientific thought due to the presence of such features for the cluster organization of production: the variety of resources types, the interdependence of the use of resources within the production chains, the heterogeneity of the territorial location of production, etc. Optimization of spatial location of enterprises within the framework of the cluster initiative has a positive impact on the efficiency of expanded reproduction of cluster products, reduces the amount of initial investment and further logistics costs for the production and transportation of raw materials and finished products.

Strategic mistakes made in the planning of placement and further development of cluster production retain their negative impact on the effectiveness of the implementation of the cluster initiative in the long term. The need to reduce the probability of errors in the planning of the processes for formation and

development of regional industrial clusters actualizes the task of improving methodological approaches to economic and mathematical modeling of production placement, production planning, optimization of production and implementation relations between the participants of clusters.

## 2. METHODOLOGICAL SPECIFICITY OF OPTIMIZATION PROBLEMS OF SPATIAL DISTRIBUTION IN CLUSTERS

The objectivity of the economic and mathematical model of spatial distribution and long-term development of production in the innovative cluster and the possibility of its practical application depends on the completeness and reliability of the underlying information. Accordingly, in the formulation of the problem should be the following general and sectoral factors of the processes of cluster development:

- own resource base of the cluster, characterized by the volume of reserves of raw materials for the main types of industrial production of the cluster;
- current availability of production facilities within the cluster territory and their location structure;
- production opportunities for further economic development of the cluster enterprises, limited by their provision with various types of resources (labor, energy, etc.);
- features of spatial distribution of enterprises and main consumers of cluster products, the nature of existing transport links, which determines the territorial structure of production and implementation processes;
- the need of the regional and national market for the main types of finished products of the cluster enterprises used in the production of raw materials and materials, as well as the objective possibilities of their export;
- the volume of investment resources planned for the creation and development of industrial production, the creation of innovative, engineering and social infrastructure.

Thus, when improving the strategic plans of spatial distribution and long-term development of production in the framework of regional industrial clusters on the basis of economic and mathematical modeling, the optimal:

- structure of production and the size of the cluster;
- logistics chains of production, transportation and sales.
- the long-term program of development of production and volume of industrial output within the consolidated group the main range of the cluster.
- optimal volumes of industrial production (enterprises of the second group) for each territory of the cluster;
- planned parameters of development of functioning and creation of new industrial enterprises of the second group for each territory of the cluster;
- the optimal transport plan between suppliers of raw materials, forming the resource base of the cluster, respectively  $(A_i, i=1,2,\dots,n)$ , and its consumers, respectively  $(B_j, j=1,2,\dots,m; C_k, k=1,2, \dots, r)$ ;
- optimal transport plan between manufacturers of industrial products of the cluster, respectively  $(B_j, j=1,2,\dots,m)$ , and its consumers, respectively  $(C_k, k=1,2, \dots,r)$ .

The distribution of activities in the regional territory and the content of these activities are very relevant to guarantee the internal cohesion of the territory (Pereira Sánchez A., Vence Deza X., 2017). At the same time, the mathematical formulation of the complex problem of optimizing the spatial distribution of production within the cluster, in the form presented above, is quite complex and voluminous for practical application. In this regard, the study initially developed a basic model for the formation and development of the cluster, based on transport algorithms as the most simple methods for optimizing the placement and development of production [17, 7, 9, 15]. The model considered below is the basis of more complex models of cluster development of the territory, including an expanded set of economic factors [1, 16]. True social responsibility is to achieve a balance between the individual, social and all expenses of humanity and the production capacity of the planet [5, 14].

### 3. FORMULATION OF THE PROBLEM FOR OPTIMAL SPATIAL DISTRIBUTION AND LONG-TERM DEVELOPMENT OF PRODUCTION IN THE CLUSTER

Consider a regional industrial cluster in the form of an ordered economic system, which includes enterprises and end users of finished products. Accordingly, among all the economic entities of the cluster can be consistently identified the following interrelated groups. The first group includes industries that form the raw material base of cluster development (including mining and primary processing). The cluster enterprises within this group preferably gravitate to the territories of the location of resources and transport networks, the production belonging to the first group is designated as  $A_i, i=1,2,\dots,n$ .

The second group includes industrial production related to the deep processing of resources: the production of components and finished products. The enterprises of this group, in contrast to the productions belonging to the first group, do not have a pronounced binding to the resource base. Production related to the second group is denoted as follows:  $B_j, j=1,2,\dots,m$ . The third group includes various types of consumers of production of the cluster of both groups. Consumers belonging to the third group are denoted as  $C_k, k=1,2, \dots, r$ .

To solve the problem you need to determine:

- optimal volume of production of raw materials for each field / territory forming the resource base of the cluster development;
- the planning parameters for the development of a functioning and establishment of new mining enterprises on the basis of deposits/territories that form the resource base of the cluster development;

Accordingly, for the proposed formulation of the problem it is necessary to add the following initial data to the model:

- the current production volumes of raw materials and the minimum allowable volumes of production expansion for each field/territory forming the resource base of the cluster development, we designate them as  $\underline{a}_{di}$ , where  $d$  is the index of the raw material type,  $d=1,2,\dots,D; i=1,2,\dots,n$ ;
- the maximum allowable volume of production of raw materials, denote them as  $(\overline{a}_{di})$ ;
- the current production volumes of industrial products in the functioning and the minimum allowable at the projected enterprises of the cluster belonging to the second group, we designate them as  $(\underline{b}_{hj})$ , where  $h$  is the index of the type of industrial products,  $h=1,2,\dots,H; j=1,2,\dots,m$ ;
- the maximum allowable production volumes of industrial products by cluster enterprises, we denote them as  $(\overline{b}_{hj})$ ;
- the needs of consumers of raw materials, we designate them as  $(C_{dk}, d=1,2,\dots,D; k=1,2, \dots,r)$ , and industrial products of cluster enterprises, denote them as  $(C_{hk}, h=1,2,\dots,H; k=1,2, \dots,r)$ .

We emphasize that within the framework of the proposed economic and mathematical model, it is necessary to recalculate the considered volume and cost indicators for the conditional production of the cluster. Consolidated types of conventional cluster products are taken on the basis of its economic specialization and main activities. To improve the objectivity of the model when recalculating, it is proposed to use the coefficients of the useful output or the rate of raw material costs per unit of industrial production of the cluster.

### 4. THE CRITERION OF OPTIMALITY OF THE SPATIAL DISTRIBUTION AND THE DEVELOPMENT OF ENTERPRISES CLUSTER

As part of the developed model, the given costs for the production and transportation of raw materials and industrial products of cluster (Z) enterprises are considered. We denote

them as follows:

$$Z_{dij} = z_{di} + w_{dij} + E_a \times I_i \quad (1)$$

$$Z_{dik} = z_{di} + w_{dik} + E_a \times I_i \quad (2)$$

$$Z_{hjk} = z_{hj} + w_{hjk} + E_b \times I_j \quad (3)$$

where:  $z$  – unit costs of production of raw materials and industrial products by cluster enterprises;

$w$  – logistics costs for transportation of raw materials and industrial products by cluster enterprises;

$I$  – specific investments in the production of raw materials and industrial products by cluster enterprises;

$E_a$  – industry average coefficient of efficiency of investments in the production of raw materials of the cluster;

$E_b$  – industry average efficiency coefficient of investments in the production of raw materials of the cluster.

## 5. FORMATION OF ECONOMIC AND MATHEMATICAL MODEL

To build an economic and mathematical model of optimal spatial distribution and long-term development of production within the cluster, we introduce the following designations of the desired mathematical variables:

$x_{dij}$  – production volume of  $d$ -type raw materials at the  $i$ -th field / cluster territory for delivery to the  $j$ -th industrial enterprise of the cluster;

$x_{dik}$  – volume production of  $d$ -th type of raw materials on the  $i$ -th field/territory cluster for the delivery of the  $k$ -th consumer;

$x_{hjk}$  – the volume of production of  $h$ -th type of industrial products at the  $j$ -th enterprise of the cluster for the supply of  $k$ -th consumer.

Accordingly, the matrix of the desired values of variables  $x$  will have the following form:

$$\dot{X} = [x_{dij}]_{D \times n \times m}, \quad (4)$$

$$\ddot{X} = [x_{dik}]_{D \times n \times r}, \quad (5)$$

$$\ddot{X} = [x_{hjk}]_{H \times m \times r} \quad (6)$$

The obtained matrices of the required values of variables  $x$  will include the optimal volumes of production of raw materials in the fields/territories of the cluster, the volume of industrial production of the cluster enterprises and supply to consumers. Thus, the formulated equation of the objective function  $F(x)$ , describing the total production and transport costs in the whole cluster will be as follows:

$$F(x) = \sum_{d,i,j}^{D,n,m} Z_{dij} x_{dij} + \sum_{d,i,k}^{D,n,r} Z_{dik} x_{dik} + \sum_{h,j,k}^{H,m,r} Z_{hjk} x_{hjk} \rightarrow \min \quad (7)$$

## 6. LIMITATIONS OF THE VALUES OF THE REQUIRED VARIABLES OF CLUSTER DEVELOPMENT

First of all, all required variables  $x$  are subject to the condition of non-negativity of values, respectively  $x \geq 0$ . We will also consider the limitations due to the economic characteristics of the location and development of production within the cluster. The restrictive conditions for the cluster's re-production facilities belonging to the first group will initially have the following form:

$$\underline{a}_{di} \leq x_{di} = \sum_{j=1}^m x_{dij} + \sum_{k=1}^r x_{dik} \leq \overline{a}_{di}; \quad \begin{cases} d = \overline{1, D} \\ i = \overline{1, n} \end{cases} \quad (8)$$

where  $x_{di}$  – the optimal volume of production of  $d$ -type raw materials at the  $i$ -th field / cluster territory.

This restriction within the framework of the proposed model is interpreted as follows: the optimal volume of production of the  $d$ -type of raw materials at the  $i$ -th field/territory of the cluster is equal to the total volume of deliveries to the cluster enterprises and end users, while the total volume should not be less than the achieved (minimum) and not more than the maximum permissible values. Accordingly, the optimal volume of production of all types of raw materials in the  $i$ -th field / cluster area will be as follows:

$$\underline{A}_i \leq x_i = \sum_{d=1}^D x_{di} \leq \overline{A}_i, \quad i = \overline{1, n} \quad (9)$$

$\underline{A}_i$  and  $\overline{A}_i$  represent the achieved (minimum allowable) and maximum allowable volumes of production of raw materials in the  $i$ -th field/cluster area and are calculated as follows:

$$\underline{A}_i = \sum_{d=1}^D \underline{a}_{di}; \quad \overline{A}_i = \sum_{d=1}^D \overline{a}_{di} \quad (10)$$

The restrictive conditions for industrial production of the cluster belonging to the second group will initially have the following form:

$$\underline{b}_{hj} \leq x_{hj} = \sum_{k=1}^r x_{hjk} \leq \overline{b}_{hj}; \quad \begin{cases} h = \overline{1, H} \\ j = \overline{1, m} \end{cases} \quad (11)$$

Herewith:

$$\sum_{k=1}^r x_{hjk} = \sum_{d_h=1}^{D_h} x_{d_h ij} = \sum_{d=1}^D x_{dij} \quad (12)$$

where:  $d_h \in \{D_h\}$  and for  $\forall i$   $x_{d_h ij}$  – the necessary volumes of production of  $d_h$ -th type of raw materials necessary for the production of industrial products necessary for consumers of the cluster  $h$ ;  $d_h = 1, 2, \dots, D_h$ .

This restriction within the framework of the proposed model is interpreted as follows: the optimal volume of production of the  $h$ -th type of industrial products at the  $j$ -th enterprise of the cluster is equal to the total volume of deliveries to end users and the volume of raw materials received from all fields/territories of the cluster. At the same time, the total

volume of industrial production should not be less than the achieved (minimum permissible) and not more than the maximum permissible values. Accordingly, the optimal volume of production of all types of industrial products by the  $j$ -th enterprise of the cluster will be as follows:

$$\underline{B}_j \leq x_j = \sum_{h=1}^H x_{hj} \leq \overline{B}_j, j = \overline{1, m} \quad (13)$$

$\underline{B}_j$  and  $\overline{B}_j$  represent the achieved (minimum permissible) and maximum permissible volumes of industrial production at the  $j$ -th enterprise of the cluster and are calculated as follows:

$$\underline{B}_j = \sum_{h=1}^H \underline{b}_{hj}; \overline{B}_j = \sum_{h=1}^H \overline{b}_{hj} \quad (14)$$

The restrictive conditions for the production of raw materials and industrial products of the cluster in order to meet the needs of all consumers of the cluster will initially be as follows:

a) for the production of cluster raw materials:

$$\sum_{i=1}^n x_{dik} = C_{dk}; \begin{cases} d = \overline{1, D} \\ k = \overline{1, r} \end{cases} \quad (15)$$

b) for the production of industrial products of the cluster:

$$\sum_{j=1}^m x_{hjk} = C_{hk}; \begin{cases} h = \overline{1, H} \\ k = \overline{1, r} \end{cases} \quad (16)$$

At this stage, the economic and mathematical model of optimization of spatial distribution and long-term development of cluster enterprises can be transformed to solve a multi-product transport problem and can be solved with the help of specialized software packages. Adaptation of the considered economic and mathematical model to the problems of formation and development of clusters is considered on the basis of conditional consideration of each enterprise of the cluster belonging to the first and second groups, as two independent economic entities  $x^1$  and  $x^2$ .

Accordingly, for cluster productions belonging to the first group, the formulation of the required variables  $x_{dij}$  and  $x_{dik}$  will take the following form:

$$x_{dij} = x_{dij}^1 + x_{dij}^2; x_{dik} = x_{dik}^1 + x_{dik}^2 \quad (17)$$

Then the restrictive conditions for the distribution of the volumes of production of raw materials already achieved in the fields/territories of the cluster will be as follows:

$$x_{di}^1 = \sum_{j=1}^m x_{dij}^1 + \sum_{k=1}^r x_{dik}^1 = \underline{x}_{di}; \begin{cases} d = \overline{1, D} \\ i = \overline{1, n} \end{cases} \quad (18)$$

The restrictive conditions of the volumes of perspective development in the fields / territories of the cluster of production of raw materials will be as follows:

$$x_{di}^2 = \sum_{j=1}^m x_{dij}^2 + \sum_{k=1}^r x_{dik}^2 \leq (\overline{a}_{di} - \underline{a}_{di}); \begin{cases} d = \overline{1, D} \\ i = \overline{1, n} \end{cases} \quad (19)$$

The difference  $(\overline{a}_{di} - \underline{a}_{di})$  describes the possible volumes of

production of raw materials in the process of cluster development, the clarification in the framework of the model also need to convert to having the solution to form:

$$\sum_{j=1}^m x_{dij}^2 + \sum_{k=1}^r x_{dik}^2 + x_{di(m+1)} = (\overline{a}_{di} - \underline{a}_{di}); \begin{cases} d = \overline{1, D} \\ i = \overline{1, n} \end{cases} \quad (20)$$

where  $x_{di(m+1)}$  – the additional required variables introduced into the model that characterize the reserve for the long – term development of the production of raw materials within the framework of cluster policy.

By analogy, the restrictive conditions for the distribution of production volumes of finished products by enterprises of the cluster will be as follows:

$$\sum_{i=1}^n (x_{dij}^1 + x_{dij}^2) + \widetilde{x}_{hj} = \underline{b}_{hj}; \begin{cases} h = \overline{1, H} \\ j = \overline{1, m} \end{cases} \quad (21)$$

$$\sum_{k=1}^r x_{hjk}^1 = \underline{b}_{hj}; \begin{cases} h = \overline{1, H} \\ j = \overline{1, m} \end{cases} \quad (22)$$

$$\sum_{k=1}^r x_{hjk}^2 + \widetilde{x}_{hj} = (\overline{b}_{hj} - \underline{b}_{hj}); \begin{cases} h = \overline{1, H} \\ j = \overline{1, m} \end{cases} \quad (22)$$

where  $\widetilde{x}_{hj}$  – are additional required variables introduced into the model that characterize the reserve for the long-term development of industrial production within the framework of cluster policy.

And further analogies restrictive conditions for all consumers of raw materials and industrial products of the cluster will initially have the following form:

a) for consumers of the cluster's raw materials:

$$\sum_{i=1}^n (x_{dik}^1 + x_{dik}^2) = C_{dk}; \begin{cases} d = \overline{1, D} \\ k = \overline{1, r} \end{cases} \quad (23)$$

b) for consumers of industrial products of the cluster:

$$\sum_{j=1}^m (x_{hjk}^1 + x_{hjk}^2) = \underline{C}_{hk}; \begin{cases} h = \overline{1, H} \\ k = \overline{1, r} \end{cases} \quad (24)$$

## 7. DISCUSSION

The most common in the scientific literature interpretation of the spatial aspect for cluster development is the classical approach of M. Porter [13], based on the neighborhood and geographical proximity of the economic entities of the cluster. M. Porter considers the cluster as a geographically concentrated system of interconnected enterprises, specialized suppliers, service firms in related activities, universities, industry development institutions, both competing and leading joint economic activities. At the same time, consideration of the scientific category "cluster" within the extended interdisciplinary approach allows to draw the following conclusions. In General, the individual structural elements

forming clusters considered in the framework of an interdisciplinary approach demonstrate, on the one hand, a rather uneven distribution in space, on the other hand, retain the necessary degree of concentration as a key attribute characteristic of the cluster.

In the framework of the subject of this study the approaches to economic and mathematical regulation presented in the works of N. Apatova [2], L. Afraymovich [1], A. Granberg[6], P. Korobova[9], T.Tatochenko, E. Toroptseva [16]. The advantage of the proposed approach to economic and mathematical modeling of cluster development processes is the shift of the research focus from separate planning within the production at separate stages of production chains or within narrow industry segments to an integrated approach. At the same time, the measurement points of economic indicators characterizing the processes of clustering of the economic space of the territory correspond to the General scientific properties of the cluster: they are, as a rule, also discrete and heterogeneously distributed. Consequently, the objectivity and reliability of the results of the study of the spatial development of regional industrial clusters is largely determined not only by the quality but also by the number of initial observations, as well as processing methods and approaches to the interpretation of the obtained values of economic indicators.

Separate solution of problems on optimization of various aspects for cluster development requires further balancing of quantitative results obtained in the course of solving individual problems. At the same time, it should be noted that, unlike the region, all stages of production within the cluster concept, as well as the existing logistics links between them are interrelated and interdependent, which leads to the consideration of the problem of optimizing the spatial location and long-term development of the cluster in the form of a single integrated model. Accordingly, the optimal quantitative values for solving the problem of spatial distribution and long-term development of cluster production are proposed to be obtained through the use of mathematical methods of quantitative analysis in the framework of application software packages [10. 15].

Also an important contribution to the methodological basis of the proposed approach to spatial modeling of clusters amounted to systematization of methods of geostatistics, conducted by V. Demyanov and E. Savelyeva [4], J. Maturana, M. Kanevsky [8], R. V. Arutyunyan [3]. Note that the main methodological limitation of the objectivity of the research of a of economic space clustering, is the presence within the boundaries of the study site areas not covered by measuring points, values of investigated indicators of cluster development. Obtaining a quantitative assessment of the studied economic indicators of cluster development at the point of space X, which was not measured, based on the available set of quantitative data for other points of space is based on the interpolation of available values. It should be noted that the interpolation of the processes of socio-economic development of the territories to date has not received a broad theoretical understanding in the scientific literature.

## 8. CONCLUSION

Thus, the practical application of the developed economic and mathematical model is possible in the form of a step-by-step solution of a multi-product transport problem with the help of specialized software packages. Economic and mathematical modeling of innovative cluster development processes complements the existing methodological tools of cluster policy, reducing the impact of subjective factors in solving problems of spatial development of productive forces.

The results obtained in the framework of the study led to the conclusion that in the modern practice of the implementation of cluster policy in the spatial localization of the formed innovative clusters, a fairly simplified approach is maintained. Within the framework of this approach, the territory of the processes of cluster development, implementation of multiplicative effects and innovative transformation of the economy often by default coincides with the existing administrative boundaries of the regions and municipalities. Therefore, to date, the development of methods of modeling and visualization of the processes of cluster development within the economic space of the regions remains relevant for economic thought. The solution of this scientific problem involves the study of the processes of transformation of the internal structure of the economic space of the region under the influence of clustering processes.

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