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Parametric Study of Design of Retaining Walls by a Derivative-Free Algorithm

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Abstract

Nowadays, it is imperative to build low-cost structures that meet the established design conditions. For this, optimizing the design while minimizing weight is a promising option. In this work, a parametric analysis is performed in the optimization of the design (weight minimization) of retaining walls for typical intervals of surcharge load, backfill slope and internal friction angle. To carry out the optimization process, a derivative-free algorithm called Modified Simulated Annealing Algorithm (MSAA) was used. These algorithms have the ability to efficiently explore multimodal and multidimensional search spaces. In addition, they have mechanisms that prevent them from being trapped in local optimum. The results indicated that the increase in the surcharge load and backfill slope led to the need for heavier structures.

Keywords: optimization, metaheuristic, retaining wall

I. INTRODUCTION

The optimization of retaining walls has become a topic of great interest by the scientific community, since its use provides preliminary design solutions that can be refined with additional analyzes for implementation in the real world [1], allowing to obtain quasi-optimal structural solutions.

The design of retaining walls is an iterative process in which different options are tested until meeting the design standards. This is generally achieved with a limited number of tests, given the high number of calculations that the designer must perform. The optimization application based on metaheuristic algorithms (derivative-free algorithm) allows this design process to be carried out automatically and iteratively, performing a high number of tests in reasonable times, providing satisfactory optimal solutions.

In recent years, different metaheuristic algorithms have been used to solve this type of problem, for example, Simulated Annealing (SA) [2], Particle Swarm Optimization (PSO) [3,4], Harmony Search (HS) [5], Gravitational search algorithm (GSA) [6], Big Bang–Big Crunch (BB-BC) [7], Charged System Search Algorithm (CSS) [8] among others [9–11]. In all these works, benchmark problems are optimized to evaluate the performance of the algorithm. In this article, a parametric study of retaining wall design is performed for typical intervals of surcharge load, backfill slope and internal friction angle using the Modified Simulated Annealing Algorithm (MSAA) [12].

II. METHODOLOGY

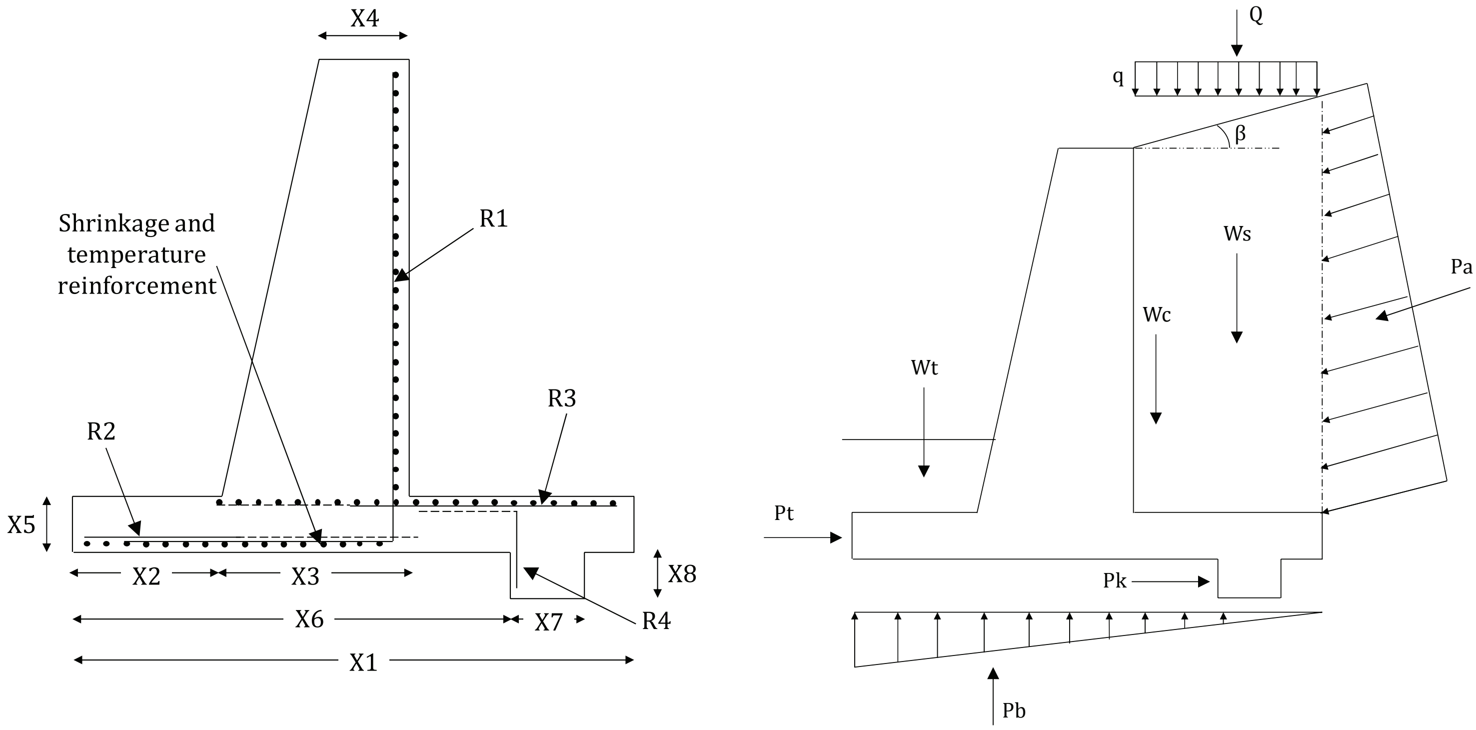
In this section, the formulation of the optimization problem and the MSAA are described.

II.I Problem formulation

To carry out the parametric study, the benchmark problem shown in Figure 1 is studied. The retaining wall has 12 design variables: X1 is the width of the base; X2 is the toe projection; X3 is the thickness at the bottom of the stem; X4 is the thickness at the top of the stem; X5 is the thickness of base slab; X6 is the distance from toe to the front of the base shear key; X7 is the width of the key; and X8 is the depth of the key. There are four additional design variables related to the steel reinforcement of the various sections of the retaining wall: R1 is the vertical steel reinforcement in the stem; R2 is the horizontal steel reinforcement in the toe; R3 is the horizontal steel reinforcement in the heel; and R4 is the vertical steel reinforcement in the base shear key.

The variables X1-X8 represent the geometry of the wall, and the variables R1-R4 represent the reinforcement. Variables X1-X8 are defined as continuous, while variables R1-R4 are discrete variables, as shown [7]. The wall design is divided into two stages: (i) geotechnical design and (ii) structural design. In the first step of the procedure the overturning, sliding, and bearing stress of the wall must be verified. In the second stage, the values of the shear force and the moment in the stem, toe, heel, or key must be verified.

Fig 1(b) shows the general forces acting on the retaining wall. According to [7], Wc is the combined weight of all the sections of the reinforced concrete wall; Ws is the weight of backfill acting on the heel; Wt is the weight of soil on the toe; Q is the surcharge load; Pa is force resulting from the active earth pressure; Pk and Pt are the forces resulting from passive earth pressure on the base shear key and front part of the toe section, respectively; and Pb is the force resulting from the bearing stress of the base soil.



(a) (b)

Fig 1. (a) Design variables for reinforced concrete cantilever retaining wall; (b) Forces acting on a cantilever retaining wall

Table 1 summarizes the formulation of the problem and Table 2 the objective function and constraints for the problem. Table 3 shown the input parameters for Problem 1 and Problem 2, respectively. For the Problem 1, a base shear key is not included in the design variables. The Problem 2 is analyzed without a base shear key and with a base shear key.

Table 1. Formulation of the problem

Equation	**#**	**Definition**
$FS_O = \frac{\sum M_R}{\sum M_O}$	(1)	FS_O→Factor of safety for overturning $\sum M_R$→sum of the moments about the toe resisting overturning. $\sum M_O$→sum of the moments about the toe tending to overturn the structure
$Ka = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\emptyset}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\emptyset}}$ $Kp = \tan^2\left(45 + \frac{\emptyset}{2}\right)$	(2)	Ka→active earth pressure Kp→passive earth pressure β→backfill slope θ→friction angle of the backfill soil
$FS_S = \frac{\sum F_R}{\sum F_D}$	(3)	FS_S→factor of safety against sliding $\sum F_R$→sum of the horizontal resisting forces $\sum F_D$→sum of the horizontal sliding forces
$\sum F_R = \left(\sum W_{wall}\right) \tan\left(\frac{2\emptyset_{base}}{3}\right) + \frac{2BC_{base}}{3} + P_p$ $\sum F_D = P_a \cos\beta$ $P_p = \frac{1}{2}\gamma_{base} D_1^2 k_p + 2c_{base} D_1 \sqrt{k_p}$	(4)	$\sum W_{wall}$→total weight of the wall $\emptyset_{base}$→internal friction of the base soil B→total width of base slab C_{base}→adhesion between the soil and the base slab γ_{base}→unit weight of the base soil D_1→total depth of the passive earth pressure block

Equation	#	Definition
$FS_B = \frac{q_u}{q_{max}}$	(5)	FS_B→factor of safety for the bearing capacity q_u→ultimate bearing capacity of the base soil
$q_{\substack{min\\max}} = \frac{\sum V}{B}\left(1 \mp \frac{6e}{B}\right)$ $e = \frac{B}{2} - \frac{\sum M_R - \sum M_O}{\sum V}$	(6)	$q_{\substack{min\\max}}$→bearing stresses on the toe and heel sections B→width of the base $\sum V$→sum of the vertical forces (resulting from the weight of wall, the soil above the base, and surcharge load) e→eccentricity of the resultant force system
$M_n = \emptyset f_y A_s \left(d - \frac{a}{2}\right)$	(7)	M_n→flexural strength $\emptyset$→nominal strength coefficient (0.9) A_s→cross-sectional area of steel reinforcement f_y→yield strength of steel d→ distance from com- pression surface to the centroid of tension steel a→depth of stress block
$V_n = \emptyset 0.17\sqrt{f_c}bd$	(8)	V_n→shear strength $\emptyset$→nominal strength coefficient (0.85) f_c→compression strength of concrete b→width of the section

Table 2. Objective function and constraints

Equation	#	Definition
$f_{peso} = W_{st} + 100V_c\gamma_c$	(9)	W_{st}→weight of steel per unit length of the wall V_c→volume of concrete per unit length of the wall γ_c→unit cost of concrete
$FS_O \geq FS_{Od}$	(10)	
$FS_S \geq FS_{Sd}$	(11)	
$FS_B \geq FS_{Bd}$	(12)	
$M_n \geq M_d$	(13)	
$V_n \geq V_d$	(14)	
$\rho_{min} = 0.25\frac{\sqrt{f_c}}{f_y} \geq \frac{1.4}{f_y}$	(15)	
$\rho_{max} = 0.85\beta_1\frac{f_c}{f_y}\left(\frac{600}{600+f_y}\right)$	(16)	
$\begin{cases} f_c \leq 30MPa & \beta_1 = 0.85 \\ f_c > 30MPa & \beta_1 = 0.85 - \frac{0.05}{7}(f_c - 30) \geq 0.65 \end{cases}$	(17)	
$X_1 \geq X_2 + X_3$	(18)	
$X_1 \geq X_6 + X_7$	(19)	

II.II Modified simulated annealing algorithm (MSAA)

The MSAA is a single-solution metaheuristic based on the cooling of metals phenomenon and it has three main stages that differentiate it from the simulated annealing (SA). Table 4 summarizes these characteristics. For more details, see [12].

Table 3. Input parameters for Problem 1 and Problem 2 [7]

Parameter	**Symbol**	**Value**	
		Problem 1	**Problem 2**
Height of stem (m)	H	3,0	4,5
Yield strength of reinforcing steel (MPa)	f_y	400	400
Compressive strength of concrete (MPa)	f_c	21	21
Concrete cover (m)	C_b	7	7
Shrinkage and temperature reinforcement percent	ρ_{st}	0,002	0,002
Surcharge load (kPa)	q	20	30
Backfill slope (°)	β	10	0
Internal friction angle of retained soil (°)	□	36	28
Internal friction angle of base soil (°)	□'	0	34
Unit weight of retained soil (kN/m³)	γ_s	17,5	17,5
Unit weight of base soil (kN/m³)	γ_s'	18,5	18,5
Unit weight of concrete (kN/m³)	γ_c	23,5	23,5
Cohesion of base soil (kPa)	c	125	0
Depth of soil in front of wall (m)	D	0,5	0,3
Factor of safety for overturning stability	FS_{Od}	1,5	1,5
Factor of safety against sliding	FS_{Sd}	1,5	1,5
Factor of safety for bearing capacity	FS_{Bd}	3,0	1,5

Table 4. Modified simulated annealing algorithm

The starting point is selected by a preliminary exploration	$X_{PxN}=I_{PxN}X_L+rand_{PxN}(X_U-X_L)$	(20)	P→number of points (states) that are desired in the search space N→number of dimensions of the problem I_{PxN} →identity matrix of size PxN X_L→lower limit of the problem X_U→upper limit of the problem $rand_{PxN}$ →matrix of random numbers (pure randomness) between 0 and 1 of size PxN.
The transition from the starting point to the new point is performed by the addition of random numbers that are within the defined radius.	$R_{i+1}=R_i\cdot\alpha$	(21)	R_i→initial radius α→radius reduction coefficient
The probability of accepting a worse solution is reduced.	$P=\frac{1}{1+e^{\left(\Delta f/T\right)}}$	(22)	P→probability of accepting the new state Δf→difference of the evaluations of the function for each state T→temperature of the system e→Euler number

III. RESULTS AND DISCUSSIONS

To perform the parametric study, a series of 100 runs are executed, and the optimized designs are calculated for typical surcharge load, backfill slope and internal friction angle.

For the Problem 1, the Figure 2(a) illustrates the weight sensitivity as a function of the variation of the surcharge load, varying the surcharge load from 0 kPa to 50 kPa. The results indicate that, between the extremes, there was an increase of 28.7% in weight. Due to the increase in surcharge load, the horizontal component of the active thrust is increased, as a result a greater geometry of the structure proportional to the surcharge load is needed to ensure sliding stability. The same happens when the backfill slope is increased for a fixed surcharge load as shown in Figure 2(b). With the increases in the slope of the embankment, there is an increase of 4.7% in the weight of the wall. However, between 0-10 ° there was a decrease in weight because the vertical as well as horizontal component of the buoyancy caused them to generate the necessary safety factors with a minimum decrease in the components. This effect occurs up to 10 °. After this value, the increase in weight is proportional to the increase in the slope. Finally, Figure 2(c) shows that the weight of the wall decreases as the internal friction angle increases. As the friction angle increases, the grain-to-grain interaction of the soil increases, generating greater internal stability in the landfill. Consequently, less weight is needed on the wall.

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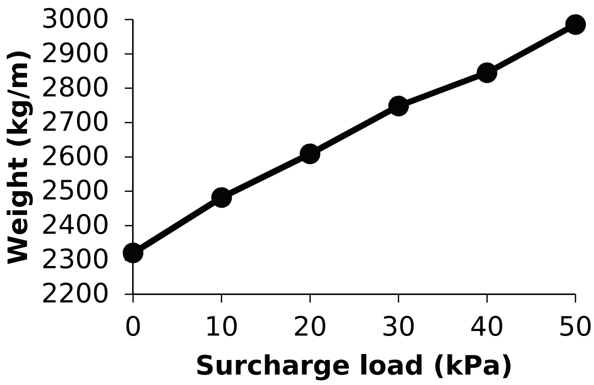
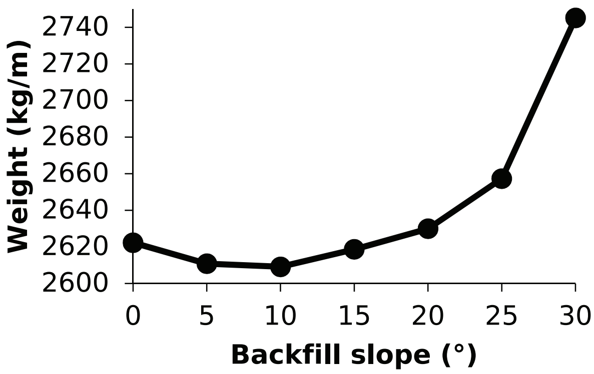


(a)



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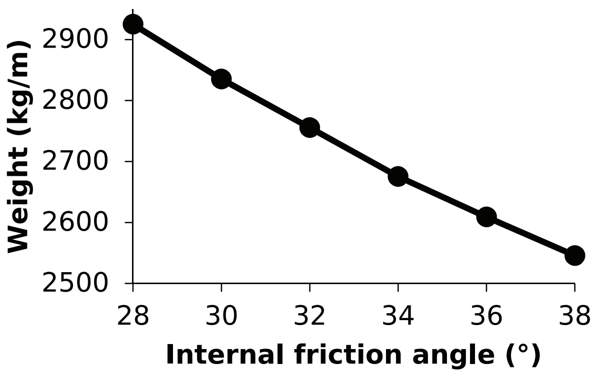

(c)

Fig. 2. (a) Effects of surcharge load; (b) backfill slope and (c) internal friction angle on weight for the Problem 1.

For the Problem 2, Figure 3(a) shows that designs with a key remain less heavy than designs without a key as the surcharge load varies. Figure 3(b) shows the designs according to the backfill slope; designs with a key are, on average, approximately 3% lighter than projects without a key. Figure 3(c) shows that for both projects, the weight of the wall decreases as the internal friction angle increases, with key designs being, on average, approximately 3% lighter than keyless designs.

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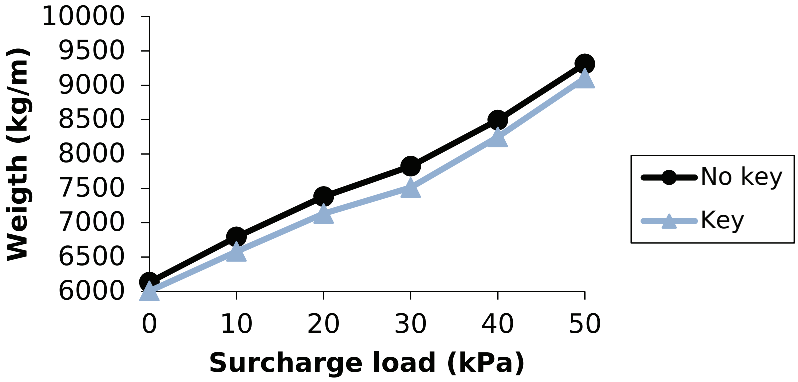

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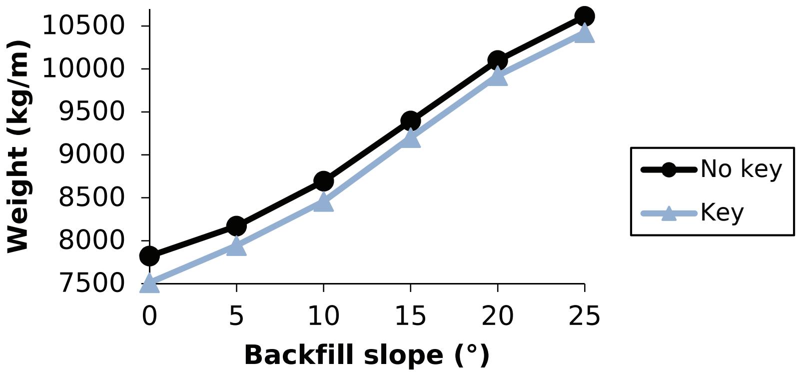

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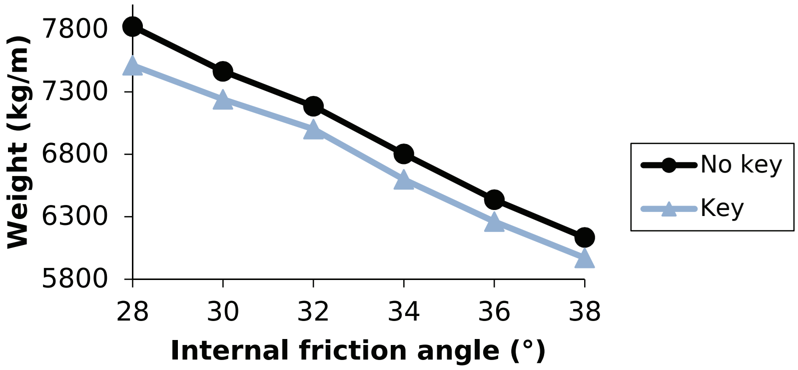

(c)

Fig. 3. (a) Effects of surcharge load; (b) backfill slope and (c) internal friction angle on weight; with and without base shear key for the Problem 2.

VI. CONCLUSION

In this work the MSAA is used to carry out a parametric study for the optimization of retaining walls (with and without key). These parameters are surcharge load, backfill slope and internal friction angle. In general, this analysis indicates that keyed retaining structures tend to be slightly more economical than those without keys for a wide range of surcharge load, backfill slope and internal friction angle.

REFERENCES

[1] Yoo DG, Kim JH, Geem ZW. Overview of Harmony Search algorithm and its applications in Civil Engineering. Evol Intell 2014;7:3–16. doi:10.1007/s12065-013-0100-4.

[2] Yepes V, Alcala J, Perea C, González-Vidosa F. A parametric study of optimum earth-retaining walls by simulated annealing. Eng Struct 2008;30:821–30. doi:10.1016/j.engstruct.2007.05.023.

[3] Khajehzadeh M, Taha MR, El-Shafie A, Eslami M. Economic design of retaining wall using particle swarm optimization with passive congregation. Aust J Basic Appl Sci 2010;4.

[4] Khajehzadeh M, Taha MR, El-Shafie A, Eslami M. Modified particle swarm optimization for optimum design of spread footing and retaining wall. J Zhejiang Univ A 2011;12:415–27. doi:10.1631/jzus.A1000252.

[5] Kaveh A, Abadi ASM. Harmony search based algorithms for the optimum cost design of reinforced concrete cantilever retaining walls. Int J Civ Eng 2011;9:1–8.

[6] Khajehzadeh M, Eslami M. Gravitational search algorithm for optimization of retaining structures. Indian J Sci Technol 2012;5:1821–7. doi:10.17485/IJST/2012/V5I1/30937.

[7] Camp C V., Akin A. Design of Retaining Walls Using Big Bang–Big Crunch Optimization. J Struct Eng 2012;138:438–48. doi:10.1061/(ASCE)ST.1943-541X.0000461.

[8] Kaveh A, Behnam AF. Charged System Search Algorithm for the Optimum Cost Design of Reinforced Concrete Cantilever Retaining Walls. Arab J Sci Eng 2013;38:563–70. doi:10.1007/s13369-012-0332-0.

[9] Sheikholeslami R, Gholipour Khalili B, Zahrai SM. Optimum Cost Design of Reinforced Concrete Retaining Walls Using Hybrid Firefly Algorithm. Int J Eng Technol 2014;6:465–70. doi:10.7763/IJET.2014.V6.742.

[10] Gandomi AH, Kashani AR, Roke DA, Mousavi M. Optimization of retaining wall design using recent swarm intelligence techniques. Eng Struct 2015;103:72–84. doi:10.1016/j.engstruct.2015.08.034.

[11] Gandomi AH, Kashani AR, Roke DA, Mousavi M. Optimization of retaining wall design using evolutionary algorithms. Struct Multidiscip Optim 2017;55:809–25. doi:10.1007/s00158-016-1521-3.

[12] Millán Páramo C, Millán Romero E. Algoritmo simulated annealing modificado para minimizar peso en cerchas planas con variables discretas. INGE CUC 2016;12:9–16. doi:10.17981/ingecuc.12.2.2016.01.