VII. CONCLUSION

In this paper, we presented an analysis of the obesity and tumor models both with and without time delay. We first used the LCIS method (Theorem 1) to find the bounds of a domain containing all compact invariant sets of models without time delay for positive values of model parameters. The generalized Gronwall Lemma applied for finding the bounds of $I(t)$ of model with time delay which is uniformly bounded. For stability of equilibrium points, we found that equilibrium points E_1, E_2, E_3, E_4 and E_5 of the models both with/without a time delay are always unstable. Next, the statement and proof of the conditions for local stability of E_6 is given in Theorem 2. Also, the conditions for the global stability of E_6 with zero-time delay are stated and proved in Theorem 3. Therefore, we analyzed the stability of the endemic equilibrium points E_8, and proved that the equilibrium by linearization was stable for the model without time delay in Theorem 4. For the time-delay model, we derived the necessary conditions for the existence of a Hopf bifurcation point (Theorem 5). In the numerical simulations, we used biologically reasonable values of parameters to test our analytical results. We found that the numerical simulations converged to the equilibrium point E_6 for choices of parameter values satisfying the conditions in Theorem 2-3. The numerical simulations also showed convergence to E_8 for time delays τ less than the Hopf bifurcation point τ_0 and limit cycle behavior for $\tau > \tau_0$ (Theorem 6). Also, the effect of some parameters on the dynamic of the model (1) are studied for the value of r_3 (the growth rate for the density of fat cells) decreases and the value of b_3 (the inverse of the carrying capacity for the density of fat) increases. It can be concluded that the obesity and time delay affect the growth of tumors. The obese people are likely to increase the density of the tumor and the obesity is a health problem of the modern world.

VIII. ACKNOWLEDGEMENT

This research is supported by the Centre of Excellence in Mathematics, the Commission on Higher Education, the Department of Mathematics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok and Kasem Bundit University, Thailand

REFERENCES

[1] S. Friberg and S. Mattson, "On the growth rates of human malignant tumors: implications for medical decision making," Journal of surgical oncology, vol. 65, no. 4, pp. 284-297, 1997.

[2] F.J. Santonja, A. Morales, R.J. Villanueva, and J.C. Cortes, "Analysing the effect of public health campaigns on reducing excess weight: A modelling approach for the Spanish autonomous region of the community of Valencia," Evaluation and program planning, vol. 35, no. 1, pp. 34-39, 2012.

[3] C. Oh and M. MA, "Optimal intervention strategies for the spread of obesity," Journal of Applied Mathematics, 2015.

[4] D. Aldila, N. Rarasati, N. Nuraini, and E. Soewono, "Optimal control problem of treatment for obesity in a closed population," International Journal of Mathematics and Mathematical Sciences, 2014.

[5] D. Laudisio, G. Muscogiuri, L. Barrea, S. Savastano, and A. Colao, "Obesity and breast cancer in premenopausal women: Current evidence and future perspectives," European Journal of Obstetrics and Gynaecology and Reproductive Biology, vol. 230, pp. 217-221, 2018.

[6] C.A.A.Rojas, M.T.Alvarez-Banuelos, J.Morales Romero, H.Suarez-Diaz, J. C. Hernandez-Fonseca, and G. Contreras-Alarcon, "Breast cancer: Metastasis, molecular subtypes, and overweight and obesity in Veracruz, Mexico," Clinical breast cancer, vol. 19, no. 1, pp. 166-171, 2019.

[7] H. Rubin, "Promotion and selection by serum growth factors drive field cancerization, which is anticipated in vivo by type 2 diabetes and obesity," Proceedings of the National Academy of Sciences, vol. 110, no. 34, pp. 13927-13931, 2013.

[8] S. D. Hursting, "Minireview: the year in obesity and cancer," Molecular endocrinology, vol. 26, no. 12, pp. 1961-1966, 2012.

[9] M. Schwab, Encyclopaedia of cancer. Springer Science and Business Media, 2008.

[10] R. A. Ku-Carrillo, S. E. Delgadillo, and B. Chen-Charpentier, "A mathematical model for the effect of obesity on cancer growth and on the immune system response," Applied Mathematical Modelling, vol. 40, no. 7, pp. 4908-4920, 2016.

[11] J. Jo, O. Gavrilova, S. Pack, W. Jou, S. Mullen, A. E. Sumner, S. W. Cushman, and V. Periwal, "Hypertrophy and/or hyperplasia: dynamics of adipose tissue growth," PLoS computational biology, vol. 5, no. 3, 2009.

[12] D. Okwan-Duodu, G. E. Umpierrez, O. W. Brawley, and R. Diaz, "Obesity-driven inflammation and cancer risk: role of myeloid derived suppressor cells and alternately activated macrophages," American journal of cancer research, vol. 3, no. 1, 2013.

[13] M. Villasana and A. Radunskaya, "A delay differential equation model for tumor growth," Journal of Mathematical Biology, vol. 47, no. 3, pp. 270-294, 2003.

[14] F. A. Rihan, D. Abdelrahman, F. Al-Maskari, F. Ibrahim, and M. A. Abdeen, "Delay differential model for tumour-immune response with chemoimmunotherapy and optimal control," Computational and math- ematical methods in medicine, 2014.

[15] P. A. Valle, K. E. Starkov, and L. N. Coria, "Global stability and tumor clearance conditions for a cancer

chemotherapy system," Communications in Nonlinear Science and Numerical Simulation, vol. 40, pp. 206-215, 2016.

[16] A. Halanay and A. Halanay, Differential equations: Stability, oscillations, time lags, vol. 6. Academic press New York, 1966.

[17] D. J. Grabiner, "Descartes' rule of signs: Another construction," The American Mathematical Monthly, vol. 106, no. 9, pp. 854-856, 1999.

[18] E. Weisstein, "Np-hard problem, mathworld a wolfram web resource," 2005.

[19] P. Liu and X. Liu, "Dynamics of a tumor-immune model considering targeted chemotherapy," Chaos, Solutions and Fractals, vol. 98, pp. 7- 13, 2017.

[20] E. W. Weisstein and C. Problem, "From mathworld a wolfram web re- source http://mathworld.wolfram. com," Mean Curvature. html, 2005.

[21] Sarud U, Sanoe K, Ekkachai K. The effect of obesity and cancer stem cells in tumor model with time delay, 9(4), COMPUSOFT, An International Journal of Advanced Computer Technology. PP. 3633-3641.