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Fuzzy ℓ-ideals Via Fuzzy Partial Ordering

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Abstract

Here we extend the theory of fuzzy ℓ-ideals via fuzzy partial ordering and develope some properties connected with fuzzy ℓ-ideals.

Key Word: Fuzzy ℓ ideals.

1 Introduction

The study of lattice struture has been very interesting and informative since its study is between the least element 0 and the greatest element 1, hence the relation between logics of algebra and modern algebra emphasized that there is a precise connection between lattice and algebraic structure. The notion of fuzzy set from the crisp set was introduced by L. A. Zadeh[4] and study of fuzzy algebraic structures was initiated by Rosenfeld, since then various algebraic structures are converted to fuzzy algebra.
Lattice structure plays a vital role in the area of theoritical computer science, communication systems and information analysis system.
Nanda.S [3] defined fuzzy lattice through fuzzy partial ordering later Kanakana chakraborty [2] has modified the definition for fuzzy lattice in different modulation.
By analysing both the papers we extend our result to fuzzy ℓ-ideal via fuzzy partial ordering.Here the ordered set are defined in terms of the upper bounds and lower bounds of subsets of the given set.
Here we define fuzzy ℓ-ideal via fuzzy partial ordering and develope some properties of fuzzy ℓ-ideal.

2 Preliminaries

Definition 2.1 *The fuzzy relation $\overline{P}$ defined over a set Λ is said to be fuzzy partial ordering if and only if it is reflexive, max-min transitive and perfectly antisymmetric.*
Let Λ be any set and let η be a fuzzy relation defined over Λ.

Then η is said to be Reflexive if $\forall a \in \Lambda, \mu_{\eta(a,a)} = 1$

Max-min transitive if $\eta.\eta \subseteq \eta$ or more explicitly if $\forall (a,b,c) \in \Lambda^3, \mu_{\eta(a,c)} \geq min\{\mu_{\eta(a,b)}, \mu_{\eta(b,c)}\}$

Perfect antisymmetric if $\forall (a,b) \in \Lambda^2, a \neq b, \mu_{\eta(a,b)} > 0 \Rightarrow \mu_{\eta(b,a)} = 0$,where $\mu_{\eta(a,b)}$ represent the membership value of the pair $(a,b) \in \eta$.

Let Λ be a fuzzy partially ordered set with a fuzzy partial order $\overline{P}$ defined over it with each $a \in \Lambda$ we associate two fuzzy sets

The dominating class $\overline{P} \geq (a)(b) = \overline{P}(b,a)$

The dominating class $\overline{P} \leq (a)(b) = \overline{P}(a,b)$

Let ω be a non fuzzy subset of η .

Then the fuzzy upper bound of ω denoted by $U_{\phi(\omega)} = \bigcap_{a \in \omega} \overline{P} \geq (a)$.

Then the fuzzy lower bound of ω denoted by $L_{\phi(\omega)} = \bigcup_{a \in \omega} \overline{P} \leq (a)$.

Definition 2.2 *Let $\overline{\tau}$ be a fuzzy partially ordered set and let $\overline{\sigma}$ be a fuzzy subset of $\overline{\tau}$. Then $\overline{\sigma}$ is said to be a fuzzy lattice in $\overline{\tau}$ if every pair of elements in $\overline{\tau}$ has a fuzzy lower bound L_ϕ and fuzzy upper bound U_ϕ, where both L_ϕ and U_ϕ are fuzzy subsets of $\overline{\tau}$ satisfying the following two conditions:*

$\mu_{max\{U_\phi\}(a)} \geq \mu_{\overline{\sigma}(a)}, \forall a \in \overline{\tau}$

$\mu_{min\{L_\phi\}(a)} \geq \mu_{\overline{\sigma}(a)}, \forall a \in \overline{\tau}$

Definition 2.3 *A non empty subset I of a ℓ attice L is called an ℓ -ideal if*

(i) x in I , y in I $\Rightarrow x \vee y$ in I .

(ii) x in I , y in L and $y \leq x \Rightarrow y$ in I .

3 Fuzzy ℓ -ideal via fuzzy partial ordering

Definition 3.1 *Let Γ be a fuzzy lattice, a fuzzy subset μ of Γ is said to be a fuzzy ℓ -ideal on Γ if it satifies the following axioms:*

(i) $\mu_{\overline{A}(0)} \geq \mu_{\overline{A}(x)}$

(ii) $\mu_{\overline{A}(x)} \leq \mu_{max\{U_\phi\}(x)}$

Example 3.1 *Let* $\mu_{\overline{A}(x)} = \{.6,.7,0\}$

$\overline{P}$	a	b	c
a	1	.8	.7
b	0	1	.9
c	0	0	1

In this case we have upper bounds as

$U_{\phi(a,b)} = \{.8,0,0\}$

$U_{\phi(b,c)} = \{.7,.9,0\}$

$U_{\phi(a,c)} = \{.7,0,0\}$

Therefore, $\mu_{max\{U_\phi\}(x)} = \{.8,.9,0\}$

$\mu_{\bar{A}(x)} \le \mu_{max\{U_\phi\}(x)}$

Thus I is a fuzzy ℓ-ideal.

Definition 3.2 *Let α and β be two fuzzy ℓ-ideal in Ω Then*

$$\mu_{\alpha(x)} = \mu_{\beta(x)} \text{ iff } \alpha = \beta \quad \forall x \in \Omega$$

$$\mu_{\alpha(x)} \subseteq \mu_{\beta(x)} \text{ iff } \alpha \subseteq \beta \quad \forall x \in \Omega$$

$$\mu_{\chi(x)} = max\{\mu_{\alpha(x)}, \mu_{\beta(x)}\} \quad \text{iff} \quad \chi = \alpha \vee \beta$$

$$\mu_{\chi(x)} = min\{\mu_{\alpha(x)}, \mu_{\beta(x)}\} \quad \text{iff} \quad \chi = \alpha \wedge \beta$$

Property 3.1 *If I is a fuzzy ℓ-ideal, $\mu_{\alpha(x)} \le \mu_{\beta(x)} \Rightarrow max\{(\mu_{\alpha(x)}, \mu_{\beta(x)}\} = \mu_{\alpha(x)}$, for all $\alpha, \beta \in I$.*

Property 3.2 *If I is a fuzzy ℓ-ideal, then $\mu_{\alpha(x)} \le \mu_{\beta(x)} \Rightarrow \mu_{\alpha(x)} - \mu_{\chi(x)} \le \mu_{\beta(x)} - \mu_{\chi(x)}$ and $\mu_{\chi(x)} - \mu_{\beta(x)} \le \mu_{\chi(x)} - \mu_{\alpha(x)}$, for all $\alpha, \beta, \chi \in I$.*

Property 3.3 *If I is a fuzzy ℓ-ideal, then, $max\{(\mu_{\alpha(x)} - \mu_{\beta(x)}), \mu_{\alpha(0)}\} + \mu_{\beta(x)} = max\{\mu_{\alpha(x)}, \mu_{\beta(x)}\}$ for all $\alpha, \beta \in I$.*

Property 3.4 *Let $\overline{P}$ be a fuzzy relation. Then $\overline{P}$ is a fuzzy ℓ-ideal if $\mu_{max\{U_\phi\}(x)} \ge \mu_{\bar{A}(x)}$*

Theorem 3.1 *If I_1 and I_2 are fuzzy ℓ-ideals of a fuzzy lattice Γ, then $max\{\mu_{I_1}, \mu_{I_2}\}$ is a fuzzy ℓ-ideal.*

Theorem 3.2 *If I_1 and I_2 are fuzzy ℓ-ideals of a fuzzy lattice Γ then $min\{\mu_{I_1}, \mu_{I_2}\}$ is a fuzzy ℓ-ideal.*

Theorem 3.3 *If $I(L)$ is the set of all fuzzy ℓ-ideals of a lattice Γ then $I(L)$ is a fuzzy lattice.*

Proof: Claim (i) $I(L)$ is reflexive, perfectly anti symmetric, max-min transitive.
Claim (ii) $max\{\mu_{I_1}, \mu_{I_2}\}$ is the ℓ.u.b of I_1 and I_2.
$min\{\mu_{I_1}, \mu_{I_2}\}$ is the g.ℓ.b of I_1 and I_2.

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